DECEMBER 2022 **P/ID 17604/PCAE1**

Time : Three hours Maximum : 80 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. From the conjunction of P : It is raining today; Q : There are 20 tables in this room.

2. Construct the truth table for $P \vee \neg Q$.

3. Give S = {2, a, {3}, 4} and R = {2, {a}, 3, 4}. Say True or False.

   (a) {a} $\in S$

   (b) $R = S$

4. Show that $A \subset A \cup B$ and $A \cap B \subset A$.

5. Give an example of a relation which is neither reflexive nor irreflexive.

6. Find the zeros of the semigroups $(P(X), \cap)$ and $(P(X), \cup)$ where X is any given set and $P(X)$ is its power set.

7. If $(G, *)$ is an ableian group, then for all a, $b \in$ G show that $(a * b)^n = a^n * b^n$.

8. Show that ({1,4,13,16}, X_{17}) is a subgroup of $(\mathbb{Z}_{17}^{*}, X_{17})$.

9. Show that a root of $x^3 - 6x - 13 = 0$ lies between 3 and 4.

10. Find L and U of the equations $2x + y = 4$ and $x + 5y = 11$.

11. Write trapezoidal rule.

12. Given $y' = x + y,\ y(0) = 1$ find $y(0.1)$ by Euler's method.

PART B — (5 × 6 = 30 marks)

Answer any FIVE questions.

13. Show that $R \to S$ can be derived from the premises $P \to (Q \to S)$, $\neg R \vee P$, and Q.

14. Prove that $A \times (B \cap C) = (A \times B) \cap (A \times C)$.

15. Let R denote a relation on the set of ordered pairs of positive integers such that $\langle x, y \rangle R \langle u, v \rangle \Leftrightarrow xv = yu$. Show that R is an equivalence relation.

16. For any commutative moniod (M, *), prove that the set of idempotent elements of M forms a submonoid.

17. Prove that S_3 is a group under composition of functions.

18. Find the real root of the equation $\cos x = 3x - 1$ correct to 3 decimal places using iteration method.

19. Dividing the range in to 10 equal parts, find the approximate value of $\int_0^{\pi} \sin x \, dx$ by (a) Trapezoidal rule (b) Simpson's rule.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

20. Show that $(x)(P(x) \vee Q(x)) \Rightarrow (x)P(x) \vee (\exists x)Q(x)$.

21. Let $f : X \to Y$ and $g : Y \to X$. Prove that $g = f^{-1}$ if and only if $g \circ f = I_X$ and $f \circ g = I_Y$.

22. Show that every finite group of order n is isomorphic group of degree n.

23. Solve the equations $27x + 6y - z = 85$; $6x + 15y + 2y = 72$; $x + y + 54z = 110$ by Gauss-Seidel method.

24. Apply Runge-Kutta method of fourth order to find the approximate value of y when $x = 0.2$ given that $y' = 3x + \frac{1}{2}y$, $y(0) = 1$.