MAY 2023 **P/ID 17604/PCAE1**

Time : Three hours Maximum : 80 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Translate into symbolic form the statement: Jack and Jill went up the hill.

2. Construct the truth table for $p \wedge \neg q$.

3. Give the power set of $\{1, \phi\}$.

4. Show that $A \subset B \Leftrightarrow A \cup B = B$.

5. Give an example of a relation which is both symmetric and antisymmetric.

6. Is the set $\mathbb{N}$ of natural numbers under the operation $x * y = \max\{x, y\}$ forms monoid? Justify your answer.

7. Find all subgroups of $(\mathbb{Z}_{12}, +_{12})$.

8. Find all left cosets of $\{[0], [3]\}$ in the group $(\mathbb{Z}_6, +_6)$.

9. State the formula to find the root of the equation $f(x)=0$ which is between $x=a$ and $x=b$ of regular falsi method.

10. Solve the linear system $x_1-4x_2=-2$, $x_1+x_2=7$ by Jordan method.

11. State Simpson's one-third rule.

12. Write down Euler algorithm to solve the differential equation $\frac{dy}{dx}=f(x,\,y)$.

PART B — ($5 \times 6 = 30$ marks)

Answer any FIVE questions.

13. Show that

$((p\vee Q)\wedge\neg(\neg P\wedge(\neg Q\wedge\neg R)))\vee(\neg P\wedge\neg Q)\vee(\neg P\wedge\neg R)$ is a tautology.

14. Prove that $A\times(B\cup C)=(A\times B)\cup(A\times C)$.

15. Let $R=\{\langle 1,\,2\rangle,\langle 3,\,4\rangle,\langle 2,\,2\rangle\}$ and $S=\{\langle 4,2\rangle,\langle 2,5\rangle,\langle 3,1\rangle,\langle 1,3\rangle\}$. Find $R\circ S$, $S\circ R$, $R\circ(S\circ R)$, $(R\circ S)\circ R$, $R\circ R$, $S\circ S$, $R\circ R\circ R$.

16. Let $(M, *)$ be a monoid. Then prove that there exists a subset $T \subset M^M$ such that $(M, *)$ is isomorphic to the monoid $(T, \circ)$.

17. Let $(G, *)$ be a finite cyclic group generated by $a \in G$. If order of G is n, then prove that n is the least positive integer such that $a^n = e$.

18. Find the inverse of the matrix $\begin{bmatrix} 4 & 1 & 2 \\ 2 & 3 & -1 \\ 1 & -2 & 2 \end{bmatrix}$ by Gaussian elimination method.

19. Use Romberg's method to compute $\int_0^1 \frac{1}{1+x^2} dx$ correct to 4 decimal places. Hence deduce an approximate value of π.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

20. Obtain the principal conjunctive normal form of $(\neg P \to R) \wedge (Q \rightleftharpoons P)$.

21. Let F_x be the set of all 1 – 1 onto mappings from $\{1, 2, 3\}$ onto itself. Find all the elements of F_x and find the inverse of each element.

22. State and prove fundamental theorem of group homomorphism.

23. Find a root of the equation $x^2 - 4\sin x = 0$ by Newton's method up to four decimal places.

24. Solve using fourth order Runge-Kutta method: $\frac{dy}{dx} + \frac{y}{x} = \frac{1}{x^2}$; $y(1) = 1.0$.

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