DECEMBER 2022 **P/ID 37501/PMA1A**

Time : Three hours Maximum : 80 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Define orbit.

2. Define p-group.

3. Define R-module.

4. Define invariants of G.

5. Define index of nilpotence of T.

6. Define similar linear transformations.

7. Define companion matrix.

8. Write rational canonical form of T.

9. Define skew-symmetric matrix.

10. Define transpose of a matrix.

11. Define Sylow p-subgroup.

12. Define Jordan block.

PART B — ($5 \times 6 = 30$ marks)

Answer any FIVE questions.

13. State and prove second Sylow theorem.

14. If G and G' are isomorphic abelian groups, then prove that for every integer s, $G(s)$ and $G'(s)$ are isomorphic.

15. If $u \in V$, is such that $uT^{n_1-k} = 0$, where $0 < k \leq n_1$, then prove that $u = u_0 T^k$ for some $u_0 \in V_1$.

16. Suppose that two matrices A, B in F_n are similar in K_n, where K is an extension of F. Prove that A and B are already similar in F_n.

17. If N is normal and if $vN^k = 0$, then prove that $vN = 0$.

18. Let G be a group containing normal subgroups H and K such that $H \cap K = \{e\}$ and $H \vee K = G$. Then prove that G is isomorphic to $H \times K$.

19. If A, $B \in F_n$, prove that $tr(AB) = tr(BA)$.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

20. State and prove Cauchy's theorem.

21. Prove that two abelian groups of order p^n are isomorphic if and only if they have the same invariants.

22. Prove that there exists a subspace W of V, invariant under T, such that $V = V_1 \oplus W$.

23. Prove that the elements S and T in $A_F(V)$ are similar $A_F(V)$ if and only if they have the same elementary divisors.

24. For all A, $B \in F_n$, prove that

    (a) $(A')' = A$

    (b) $(A + B)' = A' + B'$

    (c) $(AB)' = B'A'$

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