DECEMBER 2023 **P/ID 37501/PMA1A**

Time : Three hours Maximum : 80 marks

PART A — (10 × 2 = 20 marks)

Answer any TEN questions.

1. Define action of C on X.

2. Define orbit.

3. Define R-module.

4. Define module homomorphism.

5. Define similar linear transformations.

6. Define invariant subspace.

7. Define companion matrix.

8. Define Jordan block.

9. Define transpose.

10. Define hermitian matrix.

11. Define Sylow p-subgroup.

12. Define elementary divisors.

PART B — (5 × 6 = 30 marks)

Answer any FIVE questions.

13. Let G be a group of order p^n and let X be a finite G-set. Then prove that $|X| \equiv |X_G| (\text{mod } p)$.

14. Let X be a G–set. For $x_1, x_2 \in X$, let $x_1 \sim x_2$ if and only if there exists $g \in G$ such that $gx_1 = x_2$. Then prove that ~ is an equivalence relation on X.

15. If $u \in V$, is such that $uT^{n_1 - k} = 0$, where $0 < k \leq n_1$, then prove that $u = u_0 T^k$ for some $u_0 \in V_1$.

16. Suppose that two matrices A, B in F_n are similar in K_n, where K is an extension of F. Prove that A and B are already similar in F_n.

17. If F is of characteristic 0 and if S and T, in $A_F(V)$, are such that $ST–TS$ commutes with S, then prove that $ST–TS$ is nilpotent.

18. If $T \in A(V)$ is nilpotent then prove that $\alpha_0 + \alpha_1 T + ... + \alpha_m T^m$ is invertible if $\alpha_0 \neq 0$, where the $\alpha_i \in F$.

19. If $(vT, vT) = (v, v)$ for all $v \in V$, then prove that T is unitary.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

20. State and prove first Sylow theorem.

21. Prove that two abelian groups of order p^n are isomorphic if and only if they have the same invariants.

22. If $T \in A(V)$ has all its characteristic roots in F, then prove that there is a basis of V in which the matrix of T is triangular.

23. For each $i = 1,2,...,k$, $V_i \neq (0)$ and $V = V_1 \oplus V_2 \oplus ... \oplus V_k$. Prove that the minimal polynomial of T_i is $q_i(x)^{l_i}$.

24. If $T \in A(V)$ is such that $(vT, v) = 0$ for all $v \in V$, then prove that $T = 0$.

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