MAY 2022 **P/ID 37501/PMA1A**

Time : Three hours Maximum : 80 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Define isotropy subgroup.

2. Define p-subgroup.

3. Define invariants of G.

4. Define module homomorphism.

5. Define invariant subspace.

6. Define index of nilpotence of T.

7. Define Jordan form.

8. Define companion matrix.

9. Define elementary divisors.

10. Prove that $T \in A(V)$ is unitary if and only if $TT^* = 1$.

11. State First Sylow theorem.

12. Define trace of a matrix.

PART B — ($5 \times 6 = 30$ marks)

Answer any FIVE questions.

13. Let G be a group of order p^n and let X be a finite G-set. Then prove that $|X| \equiv |X_G| (\text{mod } p)$.

14. Suppose that G is the internal direct product of $N_1, N_2, \cdots, N_n$. Then prove that, for $i \neq j$, $N_i \cap N_j = (e)$ and if $a \in N_i$, $b \in N_j$ then $ab = ba$.

15. If $T \in A(V)$ is nilpotent, then prove that $\alpha_0 + \alpha_1 T + \cdots + \alpha_m T^m$, where the $\alpha_i \in F$ is invertible if $\alpha_0 \neq 0$.

16. Suppose that two matrices *A, B* in F_n are similar in K_n, where K is an extension of F. Prove that *A* and *B* are already similar in F_n.

17. Prove that the linear transformation T on V is unitary if and only if it takes an orthonormal basis of V into an orthonormal basis of V.

18. Let X be a G-set and let $x \in X$. Then prove that $|xG| = (G : G_x)$.

19. If G and G' are isomorphic abelian groups, then prove that for every integer s, G(s) and $G'(s)$ are isomorphic.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

20. State and prove third Sylow theorem.

21. Prove that two abelian groups of order p^n are isomorphic if and only if they have the same invariants.

22. Let $T \in A(V)$ has all its characteristic roots in F, then prove that there is a basis of V in which the matrix of T is triangular.

23. For each $i = 1,2,\cdots,k$, $v_i \neq (0)$ and $V = V_1 \oplus V_2 \oplus \cdots \oplus V_k$, prove that the minimal polynomial of T_i is $q_i(x)^{l_i}$.

24. If $T \in A(V)$ is such that $(vT, v) = 0$ for all $v \in V$, then prove that $T = 0$.

———————