MAY 2023 **P/ID 37501/PMA1A**

Time : Three hours Maximum : 80 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Define isotropy subgroup.

2. Define p-group.

3. Define internal direct product.

4. Define invariants.

5. What is canonical form of matrix?

6. Define index of nilpotence of T.

7. Define Jordan form.

8. Write rational canonical form of T.

9. Define trace of a matrix.

10. Define unitary linear transformation.

11. Define characteristic polynomial of T.

12. When will you say that two symmetric matrices A and B are congruent?

PART B — ($5 \times 6 = 30$ marks)

Answer any FIVE questions.

13. State and prove second Sylow theorem.

14. Suppose that G is the internal direct product of N_1, $N_2,\ldots,N_n$, then prove that, for $i \neq j$, $N_i \cap N_j = (e)$ and if $a \in N_i$, $b \in N_j$ then $ab = ba$.

15. Prove that two nilpotent linear transformations are similar if and only if they have same invariants.

16. Suppose that two matrices A, B in F_n are similar in K_n, where K is an extension of F. Prove that A and B are already similar in F_n.

17. For A, $B \in F_n$ and $\lambda \in F$, prove that

    (a) $tr(\lambda A) = \lambda\, tr(A)$.

    (b) $tr(A + B) = trA + trB$.

18. If $T \in A(V)$ is nilpotent then prove that $\alpha_0 + \alpha_1 T + \cdots + \alpha_m T^m$ is invertible if $\alpha_0 \neq 0$, where the $\alpha_i \in F$.

19. If $(vT, vT) = (v, v)$ for all $v \in V$, then prove that T is unitary.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

20. State and prove Cauchy's theorem.

21. Prove that two abelian groups of order p^n are isomorphic if and only if they have the same invariants.

22. If $T \in A(V)$ is nilpotent, of index of nilpotence n_1 then prove that a basis of V can be found such that the matrix of T is this basis has the form

$$\begin{pmatrix} M_{n_1} & 0 & \dots & 0 \\ 0 & M_{n_2} & \dots & 0 \\ . & . & . & . \\ . & . & . & . \\ 0 & 0 & \dots & M_{n_r} \end{pmatrix}$$

where $n_1 \geq n_2 \geq \cdots \geq n_r$ and where $n_1 + n_2 + \cdots n_r = \dim_F V$.

23. Prove that the elements S and T in $A_F(V)$ are similar in $A_F(V)$ if and only if they have same elementary divisors.

24. Prove that the linear transformation T on V is unitary if and only if it takes and orthonormal basis of V into an orthonormal basis of V.

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