MAY 2024 **P/ID 37501/PMA1A**

Time : Three hours Maximum : 80 marks

PART A — (10 × 2 = 20 marks)

Answer any TEN questions.

1. Define p-subgroup.

2. Define orbit.

3. Define R-module.

4. Define invariants.

5. What is canonical form of matrix?

6. Define similar linear transformations.

7. Define Jordan block.

8. Write rational canonical form of T.

9. Define transpose.

10. Define skew-Hermitian matrix.

11. Define elementary divisors.

12. Define normalizer of H in G.

PART B — (5 × 6 = 30 marks)

Answer any FIVE questions.

13. Let X be a G-set. Then prove that G_x is a subgroup of G for each $x \in X$.

14. If G and G' are isomorphic abelian groups, then prove that for every integer $s, G(s)$ and $G'(s)$ are isomorphic.

15. If $T \in A(V)$ is nilpotent, then prove that $\alpha_0 + \alpha_1 T + \ldots \alpha_m T^m$, where the $\alpha_i \in F$ is invertible if $\alpha_0 \neq 0$.

16. Suppose that $V = V_1 \oplus V_2$, where V_1 and V_2 are subspaces of V invariant under T. Let T_1 and T_2 be the linear transformations induced by T on V_1 and V_2 respectively. If the minimal polynomial of T_1 over F is $p_1(x)$ while that of T_2 is $p_2(x)$, then prove that the minimal polynomial for T over F is the least common multiple of $p_1(x)$ and $p_2(x)$.

17. If $A, B \in F_n$, prove that $tr(AB) = tr(BA)$.

18. Prove that the number of non isomorphic abelian groups of order p^{α}, p a prime, equals the number of partitions of n.

19. If $(vT, vT) = (v, v)$ for all $v \in V$ then prove that T is unitary.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

20. State and prove third Sylow theorem.

21. Let G be a group and suppose that G is the internal direct product of $N_1, \ldots, N_n$. Let $T = N_1 \times N_2 \times \ldots N_n$. then prove that G and T are isomorphic.

22. If $W \subset V$ is invariant under T, then prove that T induces a linear transformation $\overline{T}$ on V/W defined by $(v + W)\overline{T} = vT + W$. Further, if T satisfies the polynomial $q(x) \in F[x]$, then so does $\overline{T}$. If $p_1(x))$ is the minimal polynomial for $\overline{T}$ over F and if $p(x)$ is that for T, then prove that $p_1(x) / p(x)$.

23. For each $i = 1,2,\ldots,k$, $V_i \neq 0$ and $V = V_1 \oplus V_2 \oplus \ldots \oplus V_k$, prove that the minimal polynomial of T_i is $q_i(x)^{l_i}$.

24. For all $A, B \in F_n$, prove that

(a) $(A')' = A$

(b) $(A+B)' = A' + B'$

(c) $(AB)' = B'A'$.
