MAY 2025 **P/ID 37501/PMA1A**

Time : Three hours Maximum : 80 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. State the first Sylow theorem.

2. What is the order of center of G, if G is non-abelian group of order 8.

3. Give an example of a group of order 6 which is internal direct product of two groups.

4. Define a solvable group.

5. Define a subspace which is cyclic with respect to T.

6. State the relation between the minimal polynomials of $T|_V$ and $T|_{V/W}$, where W is a T-invariant subspace of V.

7. Define a Jordan block.

8. Define a companion matrix for a polynomial.

9. Write down two matrices of size 3×3 which are symmetric.

10. Verify $(\alpha\ S)^* = \overline{\alpha}\, S^*$.

11. Define the signature of the symmetric matrix.

12. Is there any triangular matrix, whose characteristic value is 0?

PART B — (5 × 6 = 30 marks)

Answer any FIVE questions.

13. Let H be a p-group of the finite group G. Prove that $(N(H) : H) \equiv (G : H) (\text{mod}\, p)$.

14. Let H and K are normal subgroups of G. When can we say $G \cong H \times K$? Justify your answer.

15. Let G be a finite group which is an internal direct product of N and K. Prove that $G \cong T$, where $T = N \times K$.

16. For a nilpotent T. Prove that there exists a subspace W of V, invariant under T, such that $V = V_1 \oplus W$, where the terms define appropriately.

17. For a given linear transformation T, is there a way to get the matrix of the T as diagonal. Prove your statement in detail.

18. Prove that, if F is a field of characteristic 0 and if $T \in A_F(V)$ is such that $trT_i = 0$ for all $i > 1$, then T is nilpotent.

19. Prove that, if $T \in A(V)$ is such that $(vT, v) = 0$ for all $v \in V$, then $T = 0$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

20. State and prove the Cauchy's theorem.

21. Prove that two abelian groups of order p^n are isomorphic if and only if they have the same invariants.

22. If $T \in A(V)$ is nilpotent of index of nilpotence n_1, Prove that there exists a basis of V such that the matrix of T in this basis has the form.

$$\begin{bmatrix} M_{n_1} & 0 & 0 & \ldots 0 \\ 0 & M_{n_2} & 0 \ldots & 0 \\ 0 & 0 & 0 & \ldots M_{n_k} \end{bmatrix}$$

Where n_i and M_{n_i} are defined appropriately.

23. Let $T \in A_F(V)$ and $S \in A_F(W)$ are such that for any $v \in V$, $(vS)\psi = (v\psi)T$. Then prove that S and T have the same elementary divisors.

24. Prove that a Normal linear map is Unitary if and only if its characteristic roots are all of absolute value 1.
