DECEMBER 2022 **P/ID 37502/PMA1B**

Time : Three hours Maximum : 80 marks

SECTION A — (10 × 2 = 20 marks)

Answer any TEN questions.

1. Define Total Variation.

2. Write Dirichlet's test.

3. State the Additive Property of Total Variation.

4. State the theorem on Reduction of a Riemann-Stieltjes integral to a finite sum.

5. Write the Riemann's condition.

6. State Second Mean-Value Theorem for Riemann-Stieltjes Integrals.

7. Define Oscillation of f on T.

8. Define the Cauchy product of $\sum a_n$ and $\sum b_n$.

9. Define Cesaro Summable.

10. State Bernstein Theorem.

11. State Cauchy condition for uniform convergence of series.

12. Define Mean Convergence.

SECTION B — (5 × 6 = 30 marks)

Answer any FIVE questions.

13. Assume that *f* and *g* are each of bounded variation on $[a,b]$. Then prove that so are their sum, difference and product. Also, prove that

$V_{f\pm g} \le V_f + V_g$ and $V_{f\cdot g} \le AV_f + BV_g$, where $A = \sup\{|g(x)| : x \in [a,b]\}$, $B = \sup\{|f(x)| : x \in [a,b]\}$.

14. Assume that $\alpha \nearrow$ on [*a*, *b*]. If $f \in R(\alpha)$ on [*a*, *b*], then prove that $f^2 \in R(\alpha)$ on $[a,b]$.

15. State and prove the Second fundamental theorem of integral calculus.

16. State and prove the theorem on Change of variable in a Riemann integral.

17. If a series is convergent with sum *S*, then prove that it is also *(C,* 1) summable with Cesaro sum *S*.

18. Let $f(p,q) = \dfrac{pq}{p^2+q^2}$, $(p = 1,2,\ldots; q = 1,2,\ldots)$.

Find $\lim\limits_{p\to\infty}\left(\lim\limits_{q\to\infty} f(p,q)\right)$ and $\lim\limits_{p,q\to\infty} f(p,q)$, if it exist.

19. Let α be of bounded variation on [a, b]. Assume that each term of the sequence $\{f_n\}$ is a real-valued function such that $f_n \in R(\alpha)$ on [a, b] for each n = 1,2,… Assume that $f_n \to f$ uniformly on [a, b] and define

$$g_n(x) = \int_a^x f_n(t)\,d\alpha(t) \text{ if } x \in [a,b],\ n = 1,2,\ldots.$$

Then prove that

(a) $f \in R(\alpha)$ on $[a,b]$.

(b) $g_n \to g$ uniformly on [a, b], where $g(x) = \int_a^x f(t)\,d\alpha(t)$.

SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

20. Let $\sum a_n$ be an absolutely convergent series having sums s. Then prove that every rearrangement of $\sum a_n$ also converges absolutely and has sum s.

21. Assume that $f \in R(\alpha)$ on $[a, b]$ and assume that α has a continuous derivative α' on $[a, b]$. Prove that the Riemann's integral $\int_a^b f(x)\alpha'(\alpha)dx$ exists and $\int_a^b f(x)d\alpha(x) = \int_a^b f(x)\alpha'(x)dx$.

22. State and prove Lebesgue's criterion for Riemann-integrability.

23. State and prove Bernstein Theorem.

24. State and prove Dirichlet's test for uniform convergence.
