MAY 2022 **P/ID 37502/PMA1B**

Time : Three hours Maximum : 80 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Define a function of Bounded Variation. Give examples.

2. Write Abel's test.

3. Define Riemann-Stieltjes sum of f with respect to α.

4. Define step function.

5. Write the Riemann's condition.

6. State Second fundamental theorem of integral calculus.

7. Define Oscillation of f on T.

8. Define a double series.

9. Write the Cauchy condition for products.

10. State Mertens theorem.

11. State Weierstrass M-test.

12. State Arzela's theorem.

PART B — ($5 \times 6 = 30$ marks)

Answer any FIVE questions.

13. State and prove the Partial Summation formula of Abel.

14. Assume that $\alpha \nearrow$ on $[a,b]$. If $f \in R(\alpha)$ on $[a,b]$, then prove that $|f| \in R(\alpha)$ on $[a,b]$ and

$$\left| \int_a^b f(x)\, d\alpha(x) \right| \leq \int_a^b |f(x)|\, d\alpha(x).$$

15. State and prove First Mean-Value Theorem for Riemann-Stieltjes integrals.

16. If f is continuous on $[a,b]$ and if α is of bounded variation on $[a,b]$, then prove that $f \in R(\alpha)$ on $[a,b]$.

17. State and prove the theorem on Cauchy condition for products.

18. Find the $(C,1)$ sum of $\sum_{n=1}^{\infty}(-1)^{n-1}$.

19. State and prove the theorem on Cauchy condition for Uniform Convergence.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

20. Let f be of bounded variation on $[a,b]$. If $x \in (a,b]$, let $V(x) = V_f(a,x)$ and put $V(a) = 0$. Prove that every point of continuity of f is also a point of continuity of V. Prove that the converse is also true.

21. If $f \in R(\alpha)$ on $[a,b]$, then prove that $\alpha \in R(f)$ on $[a,b]$ and

$$\int_a^b f(x)\,d\alpha(x) + \int_a^b \alpha(x)dfx = f(b)\alpha(b) - f(a)\,\alpha(a).$$

22. Let $Q = \{(x, y) : a \le x \le b,\ c \le y \le d\}$. Assume that α is of bounded variation on $[a,b]$, β is of bounded variation on $[c,d]$ and f is continuous on Q. If $(x,y) \in Q$, X define $F(y) = \int_a^b f(x,y)d\alpha(x),\ G(x) = \int_c^d f(x,y)d\beta(y)$. Then prove that $F \in R(\beta)$ on $[c,d]$, $G \in R(\alpha)$ on $[a,b]$ and $\int_c^d F(y)\,d\beta(y) = \int_a^b G(x)\,d\alpha(x)$.

23. Prove Merten's theorem.

24. State and prove Dirichlet's test for uniform convergence.

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