MAY 2023 **P/ID 37502/PMA1B**

Time : Three hours Maximum : 80 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Define absolute and conditional Convergence.

2. Define rearrangement of a series.

3. Write Partial Summation Formula of Abel.

4. State Euler's summation formula.

5. Write the Riemann's condition.

6. State First Mean-Value Theorem for Riemann-Stieltjes Integrals.

7. State the theorem on Change of variable in a Riemann integral.

8. Define Double Sequence.

9. Define an infinite product.

10. State Tauber's Theorem.

11. State Arzela's Theorem.

12. Define Riemann-integrable functions.

PART B — ($5 \times 6 = 30$ marks)

Answer any FIVE questions.

13. Let f be of bounded variation on $[a,b]$. Let V be defined on $[a,b]$ as follows: $V(x) = V_f(a,x)$ if $a < x \leq b$, $V(a) = 0$. Then prove that

    (a) V is an increasing function on $[a,b]$.

    (b) $V - f$ is an increasing function on $[a,b]$.

14. If $f \in R(\alpha)$ and $g \in R(\alpha)$ on $[a,b]$, then prove that $c_1 f + c_2 g \in R(\alpha)$ on $[a, b]$ (for any two constants c_1 and c_2) and $\int_a^b (c_1 f + c_2 g) d\alpha = c_1 \int_a^b f d\alpha + c_2 \int_a^b g d\alpha$.

15. State and prove Second Mean-Value Theorem for Riemann-Stieltjes Integrals.

16. Let $\sum a_n$ and $\sum b_n$ be two absolutely convergent series with sums A and B, respectively. Let f be the double sequence defined by the equation $f(m, n) = a_m b_n$, if $(m, n) \in \mathbb{Z}^+ \times \mathbb{Z}^+$. Then prove that $\sum_{m,n} f(m, n)$ converges absolutely and has the sum AB.

17. Assume that each $a_n > 0$. Then prove that the product $\prod (1 + a_n)$ converges if and only if the series $\sum a_n$ converges.

18. State and prove Tauber Theorem.

19. Assume that $f_n \to f$ uniformly on S. If each f_n is continuous at a point c of S, then prove that the limit function f is also continuous at c.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

20. State and prove the Riemann's theorem on conditionally Convergent Series.

21. Assume that $c \in (a,b)$. If two of the three integrals in the following equality exist, then prove that the third also exists, and $\int_a^c f d\alpha + \int_a^b f d\alpha = \int_a^b f d\alpha$.

22. Let $Q = \{(x,y) : a \le x \le b,\ c \le y \le d\}$. Assume that α is of bounded variation on $[a, b]$, β is of bounded variation on $[c,d]$ and f is continuous on Q. If $(x,y) \in Q$, define $F(y) = \int_a^b f(x,y) d\alpha(x)$, $G(x) = \int_c^d f(x,y) d\beta(y)$. Then prove that $F \in R(\beta)$ on $[c,d]$, $G \in R(\alpha)$ on $[a,b]$ and $\int_c^d F(y) d\beta(y) = \int_a^b G(x) d\alpha(x)$.

23. State and prove Abel's limit Theorem.

24. State and prove Dirichlet's test for uniform convergence.

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