MAY 2024 **P/ID 37502/PMA1B**

Time : Three hours Maximum : 80 marks

SECTION A — (10 × 2 = 20 marks)

Answer any TEN the questions.

1. Define the total variation.

2. Define absolute and conditional convergence.

3. Define Rearrangement of series.

4. Define Riemann-Stieltjes integral.

5. State the sufficient condition for the existence of Riemann Stieltjes integrals.

6. State the Second Mean-Value theorem for Riemann-Stieltjes integrals.

7. Define Cesáro summable.

8. State Tauber's theorem.

9. State the Cauchy condition for infinite products.

10. State Cauchy condition for uniform convergence of series.

11. Define Pointwise convergence of sequence of functions.

12. Define convergence in mean.

SECTION B — (5 × 6 = 30 marks)

Answer any FIVE questions.

13. State and prove Dirichlet's test.

14. Assume that α is $\nearrow$ on [a, b]. Then prove that $\underline{I}(f,\alpha) = \bar{I}(f,\alpha)$.

15. If f is continuous on [a, b] and if α is of bounded variation on [a, b], then prove that $f \in R(\alpha)$ on [a,b].

16. Prove that absolute convergence of $\Pi(1+a_n)$ implies convergence.

17. Assume that $f_n \to f$ uniformly on S. If each F_n is continuous at a point c of S, then prove that the limit function f is also constant at c.

18. Let f be defined and bounded on [a,b]. For each $\varepsilon > 0$, define the set J_ε as follows:

$J_\varepsilon = \{x : x \in [a,b], w_f(x) \geq \varepsilon\}$. Then prove that J_ε is a closed set.

19. Given a power series $\sum_{n=0}^{\infty} a_n (z - z_0)^n$ let

$$\lambda = \limsup_{n \to \infty} \sqrt[n]{|a_n|}, \; r = \frac{1}{\lambda}$$

(where $r = 0$, if $\lambda = +\infty$ and $r = +\infty$, if $\lambda = 0$). Prove that the series converges absolutely if $|z - z_0| < r$ and diverges if $|z - z_0| > r$. Furthermore, prove that the series converges uniformly on every compact subset interior to the disk of convergence.

SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

20. Let f be of bounded variation on $[a,b]$. If $x \in (a,b]$, let $V(x) = V_f(a,x)$ and put $V(a) = 0$. Prove that every point of continuity of f is also a point of continuity of V. Prove that the converse also true.

21. State and prove Euler's summation formula.

22. State and prove the theorem on Lebesgue's criterion for Riemann integrability.

23. Assume that $\sum a_n (z - z_0)^n$ converges, if $z \in B(z_0; r)$. Suppose that the equation $f(z) = \sum_{n=0}^{\infty} a_n (z - z_0)^n$ is known to be valid for each

z in some open subset S of $B(z_0;r)$. For each point z_1 in S, prove that there exists a neighborhood $B(z_1;R)\subseteq S$ in which f has a power series expansion of the form $f(z)=\sum_{k=0}^{\infty} b_k(z-z_1)^n$, where $b_k=\binom{n}{k}a_n(z_1-z_0)^{n-k}$, $(k=0,1,2,\ldots)$.

24. State and prove Dirichlet's test for uniform convergence.
