DECEMBER 2022 **P/ID 37503/PMA1C**

Time : Three hours Maximum : 80 marks

SECTION A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Find the solution of $y'' + 10y = 0$, $y(0) = \pi$, $y'(0) = \pi^2$.

2. Find the solutions of $y'' + 9y = \sin 3x$.

3. Determine whether the functions $\phi_1(x) = \cos x$, $\phi_2(x) = \sin x$ are linearly dependent or independent.

4. Find the three linearly independent solutions of $y''' - 4y' = 0$.

5. Are the set of functions $\phi_1(x) = e^{ix}$, $\phi_2(x) = \sin x$, $\phi_3(x) = 2\cos x$ linearly dependent or independent.

6. Define a linear dependent function.

7. State the Existence Theorem for analytic coefficients.

8. Write the Legendre equation.

9. Write the Bessel equation of order α.

10. Find the regular singular points of Bessel equation of order α.

11. Define the exact differential equation.

12. Write the Lipschitz condition.

SECTION B — ($5 \times 6 = 30$ marks)

Answer any FIVE questions.

13. Prove that two solutions ϕ_1, ϕ_2 of $L(y) = 0$ are linearly independent on an interval I if and only if $W(\phi_1, \phi_2)(x) \neq 0$ for all x in I.

14. Let a_1, a_2 be any constants and consider the equation $L(y) = y'' + a_1 y' + a_2 y = 0$. If r_1, r_2 are distinct roots of the characteristic polynomial p, where $p(r) = r^2 + a_1 r + a_2$ then prove that the functions ϕ_1, ϕ_2 defined by $\phi_1(x) = e^{r1x}, \phi_2(x) = e^{r2x}$ are solutions of $L(y) = 0$.

15. Let $\phi_1, \ldots, \phi_n$ be n solution of $L(y) = 0$ on an interval I containing x_0. Prove that they are linearly independent on I if and only if $W(\phi_1, \ldots, \phi_n)(x_0) \neq 0$.

16. Let $\phi_1,\ldots,\phi_n$ be the n solutions of $L(y)=0$ on I satisfying $\phi_i^{(i-1)}(x_0)=1$, $\phi_i^{(j-1)}(x_0)=0$, $j\neq i$. If ϕ is any solution of $L(y)=0$ on I, prove that there are n constants $c_1,\ldots,c_n$ such that $\phi=c_1\phi_1+\ldots+c_n\phi_n$.

17. Prove that there exist n linearly independent solutions of $L(y)=0$ on I.

18. Find the indicial polynomial for $x^2y''+(x+x^2)y'-y=0$.

19. Prove that the equation $2xy\,dx+(x^2+3y^2)dy=0$ is exact and solve.

SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

20. If ϕ_1,ϕ_2 be any two solutions of $L(y)=0$ on an interval I containing a point x_0 then prove that $W(\phi_1,\phi_2)(x)=e^{-a1(x-x_0)}W(\phi_1,\phi_2)(x_0)$.

21. State and prove the uniqueness theorem linear differential equations with constants coefficients.

22. Let b be continuous on an interval I and let $\phi_1,\ldots,\phi_n$ be a basis for the solutions of $L(y)=0$ on I. Prove that every solution ψ of $L(y)=b(x)$ can be written as $\psi(x)=\psi_p+c_1\phi_1+\ldots+c_n\phi_n$, where ψ_p is a particular solution of $L(u)=b(x)$ and $c_1,\ldots,c_n$ are constants.

23. Find the solution of the Bessel equation.

24. State and prove Existence theorem for convergence of the successive approximations.
