DECEMBER 2023 **P/ID 37503/PMA1C**

Time : Three hours Maximum : 80 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Find the solution of $y'' - 2y' - 3y = 0, y(0) = 0, y'(0) = 1$.

2. Solve $y'' + w^2 y = 0$.

3. Determine whether the functions $\phi_1(x) = x, \phi_2(x) = e^{rx}$ are linearly dependent or independent.

4. Define Wronskian.

5. Are the set of functions $\phi_1(x) = e^{ix}$, $\phi_2(x) = \sin x$, $\phi_3(x) = 2\cos x$ linearly dependent or independent.

6. Define linear space of function.

7. Define the power series solution of differential equation.

8. State Bessel function of zero order of the second kind.

9. Find all real valued solution of the equation $yy'' = x$.

10. Write the Bessel equation of order α.

11. Define the exact differential equation.

12. Define convergence of the successive approximation.

PART B — ($5 \times 6 = 30$ marks)

Answer any FIVE questions.

13. State and prove the Uniqueness theorem for linear equations with constant coefficients.

14. Let ϕ_1, ϕ_2 be any two linearly independent solutions of $L(y) = 0$ on an interval I. Prove that every solution ϕ of $L(y) = 0$ can be written uniquely as $\phi = c_1\phi_1 + c_2\phi_2$, where c_1, c_2 are constants.

15. Let $\alpha_1, ..., \alpha_n$ be any n constants and let x_0 be any real number. Prove that there exists at most one solution ϕ of $L(y) = 0$ satisfying $\phi(x_0) = \alpha_1, \phi'(x_0) = \alpha_2, ..., \phi^{n-1}(x_0) = \alpha_n$.

16. Let $\phi_1, ..., \phi_n$ be the n solutions of $L(y) = 0$ on I satisfying $\phi_i^{(i-1)}(x_0) = 1, \phi_i^{(j-1)}(x_0) = 0, j \neq i$. If ϕ is any solution of $L(y) = 0$ on I, prove that there are n constants $c_1, ..., c_n$ such that $\phi = c_1\phi_1 + ... + c_n\phi_n$.

17. Let $b_1,...,b_n$ non-negative constants such that for all x in I $|a_j(x)| \le b_j$, $(j=1,...,n)$ and define k by $k = 1 + b_1 + ... + b_n$. If x_0 is a point in I and ϕ is a solution of $L(y) = 0$ on I. Prove that $\|\phi(x_0)\| e^{-k|x-x_0|} \le \|\phi(x)\| \le \|\phi(x_0)\| e^{k|x-x_0|}$ for all x in I.

18. Find the indicial polynomial for $x^2y'' + a(x)xy' + b(x)y = 0$.

19. Prove that the equation $2xy\,dx + (x^2 + 3y^2)dy = 0$ is exact and solve.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

20. Prove that two solutions ϕ_1, ϕ_2 of $L(y) = 0$ are linearly independent on an interval I if and only if $W(\phi_1, \phi_2)(x) \neq 0$ for all x in I.

21. Compute the solution ψ of $y''' + y'' + y' + y = 1$ which satisfies $\psi(0) = 0, \psi'(0) = 1$ and $\psi''(0) = 0$.

22. Let $\phi_1,...,\phi_n$ be n linearly independent solution of $L(y) = 0$ on an intervals I. If ϕ is any solution of $L(y) = 0$ on I, prove that it can be represented in the form $\phi = c_1\phi_1 + ... + c_n\phi_n$, where $c_1,...,c_n$ are constants.

23. Find the solution for Bessel equation $x^2y'' + xy' + (x^2 - \alpha^2)y = 0$ of order α.

24. Let f be a continuous real valued function on the rectangle $R : |x - x_0| \le a, |y - y_0| \le b, (a, b > 0)$ and let $|f(x, y)| \le M$ for all (x, y) in R. Suppose that f satisfies a Lipschitz condition with constant K and R. Prove that the successive approximations.

$$\phi_0(x) = y_0, \phi_{k+1}(x) = y_0 + \int_{x_0}^{x} f(t, \phi_k(t))dt, (k = 0,1,2...),$$

converge on the interval
$I : |x - x_0| \le \alpha = \min\{a, b/M\}$ to a solution ϕ of the initial value problem $y' = f(x, y), y(x_0) = y_0$ on I.

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