MAY 2022 **P/ID 37503/PMA1C**

Time : Three Hours Maximum : 80 Marks

SECTION A — ($10 \times 2 = 20$ marks)

Answer any TEN questions

1. Find the solution of
$y'' - 2y' - 3y = 0, y(0) = 0, y'(0) = 1$

2. Define non homogeneous differential equation.

3. Determine whether the functions $\phi_1(x) = x$, $\phi_2(x) = e^{rx}$ are linearly dependent or independent.

4. Find the three linearly independent solutions of $y''' - 4y' = 0$

5. Are the set of functions
$\varphi_1(x) = e^{ix}, \varphi_2(x) = \sin x, \varphi_3(x) = 2\cos x$ linearly dependent or independent.

6. Define a linear independent function.

7. Define the power series solution of differential equation.

8. Define the n-th Legendre polynomial.

9. Write the Bessel function of zero order of the first kind.

10. Write the Bessel equation of order α

11. Define the exact differential equation.

12. Define the variable separable method for first order differential equation.

SECTION B — ($5 \times 6 = 30$ marks)

Answer any FIVE questions

13. Let α, β be any two constants and let x_0 be any real number. Prove that for any interval I containing x_0 there exists at most one solution ϕ of the initial value problem $L(y) = 0, y(x_0) = \alpha, y'(x_0) = \beta$

14. Let ϕ be any solution of $L(y) = y'' + a_1 y' + a_2 y = 0$ on an interval I containing x_0. Prove that for all x in I $\|\phi(x_0)\| e^{-k|x-x_0|} \le \|\phi(x)\| \le \|\phi(x_0)\| e^{k|x-x_0|}$ where $\|\varphi(x)\| = \left[|\varphi(x)|^2 + |\varphi'(x)|^2\right]^{\frac{1}{2}}, k = 1 + |a_1| + |a_2|$.

15. Let $\phi_1, ..., \phi_n$ be n linearly independent solution of $L(y) = 0$ on an interval I. If $c_1, ..., c_n$ are any constants $\phi = c_1\phi_1 + ... + c_n\phi_n$ is a solution and every solution may be represented in this form.

16. Let $\phi_1,.....,\phi_n$ be the n solutions of $L(y)=0$ on I satisfying $\phi_i^{(i-1)}(x_0)=1,\phi_i^{(j-1)}(x_0)=0, j\neq i$. If ϕ is any solution of $L(y)=0$ on I, prove that there are n constants $c_1,...,c_n$ such that $\varphi = c_1\varphi_1 + ... + c_n\varphi_n$.

17. If $\phi_1,.....,\phi_n$ be n solution of $L(y)=0$ on an interval I, prove that they are linearly independent there if and only if $W(\phi_1...\phi_n)(x)\neq 0$ for all x in I.

18. Find the indicial polynomial for $x^2y'' + (x+x^2)y' - y = 0$

19. Prove that the equation $2xy\,dx + (x^2+3y^2)dy = 0$ is exact and solve.

SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions

20. Let ϕ_1,ϕ_2 be any two linearly independent solutions of $L(y)=0$ on an interval I and x_0 be any point on I. Prove that ϕ_1,ϕ_2 are linearly independent on I if and only if $W(\phi_1,\phi_2)(x_0)\neq 0$

21. Let ϕ be any solution of $L(y) = y^n + a_1y^{n-1} + ... + a_ny = 0$ on an interval I containing x_0. Prove that for all x in I $\|\phi(x_0)\|e^{-k|x-x_0|} \le \|\phi(x)\| \le \|\phi(x_0)\|e^{k|x-x_0|}$ where

$\|\varphi(x)\| = \left[|\varphi(x)|^2 + |\varphi'(x)|^2\right]^{\frac{1}{2}}$, $k = 1 + |a_1| + ... + |a_n|$.

22. Let x_0 be in I and let $\alpha_1,..,\alpha_n$ be any n constants. Prove that there is at most one solution ϕ of $L(y)=0$ on I satisfying $\phi(x_0)=\alpha_1,\phi'(x_0)=\alpha_2,...,\phi^{(n-1)}(x_0)=\alpha_n$

23. Find the indicial polynomial for $x^2y''+a(x)xy'+b(x)y=0$ where $a(x)=\sum_{k=0}^{\infty}\alpha_k x^k$ and $b(x)=\sum_{k=0}^{\infty}\beta_k x^k$

24. Suppose S is either a rectangle $|x-x_0|\le a,|y-y_0|\le b,(a,b>0),$ or a strip $|x-x_0|\le a,|y|<\infty,(a>0),$ and that f is a real valued function defined on S such that $\frac{\partial f}{\partial y}$ exist, is continuous on S and $\left|\frac{\partial f}{\partial y}(x,y)\right|\le K,$ for some $K>0$. Prove that f satisfies a Lipschitz condition on S with Lipschitz constants K.
