MAY 2024 **P/ID 37503/PMA1C**

Time : Three hours Maximum : 80 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Solve $y'' + \omega^2 y = 0$.

2. State Uniqueness theorem for linear equations with constant coefficients.

3. Determine whether the functions $\phi_1(x) = \cos x$, $\phi_2(x) = \sin x$ are linearly dependent or independent.

4. Find the three linearly independent solutions of $y''' - 4y' = 0$.

5. When you say that a set of functions are linearly independent?

6. Define Gramian.

7. State existence theorem for analytic coefficients.

8. Write the Legendre equation.

9. Find all real valued solutions of the equation $yy' = x$.

10. Write the regular singular points of Bessel equation of order α.

11. Define exact differential equation.

12. Define singular point.

PART B — ($5 \times 6 = 30$ marks)

Answer any FIVE questions.

13. Let ϕ_1, ϕ_2 be two solutions of $L(y) = 0$ on an interval I and x_0 be any point in I, Prove that ϕ_1, ϕ_2 are linearly independent on I if and only if $W(\phi_1, \phi_2)(x_0) \neq 0$.

14. Let ϕ_1, ϕ_2 be the two solutions of $L(y) = 0$ given by $\phi_1(x) = e^{r_1 x}$, $\phi_2(x) = e^{r_2 x}$ in case $r_1 \neq r_2$ and by $\phi_1(x) = e^{r_1 x}$, $\phi_2(x) = xe^{r_1 x}$ in case $r_1 = r_2$. Prove that c_1, c_2 are any two constants the function $\phi = c_1\phi_1 + c_2\phi_2$ is a solution of $L(y) = 0$ on $-\infty < x < \infty$.

15. Let $\alpha_1, \ldots \alpha_n$ be any n constants and let x_0 be any real number. Prove that there exists at most one solution ϕ of $L(y) = 0$ satisfying $\phi(x_0) = \alpha_1, \phi'(x_0) = \alpha_2, \ldots \phi^{n-1}(x_0) = \alpha_n$.

16. Let x_0 be in I and let $\alpha_1,\ldots\alpha_n$ be any n constants. Prove that there is at most one solution ϕ of $L(y)=0$ satisfying $\phi(x_0)=\alpha_1,\ \phi'(x_0)=\alpha_2,\ldots,\phi^{n-1}(x_0)=\alpha_n$.

17. Let $a_1,\ldots a_n$ be. continuous functions on an interval I containing the point = x_0. If $\alpha_1,\ldots\alpha_n$ are any n constants. Prove that there is a solution ϕ of $L(y)=y^{(n)}+a_1(x)y^{(n-1)}+\ldots+a_n(x)y=0$ on I satisfying $\phi(x_0)=\alpha_1,\ \phi'(x_0)=\alpha_2,\ldots,\phi^{n-1}(x_0)=\alpha_n$.

18. Find the indicial polynomial for $x^2y''+\frac{3}{2}xy'+xy=0$.

19. Prove that the equation $2xy\,dx+\left(x^2+3y^2\right)dy=0$ is exact and solve.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

20. Prove that for any real x_0 and constants α,β there exists a solution ϕ of the initial value problem $L(y)=0,\ y(x_0)=\alpha,\ y'(x_0)=\beta$ on $-\infty<x<\infty$.

21. Let $\alpha_1,\ldots,\alpha_n$ be any n constants and let x_0 be any real number. Prove that there exists a solution ϕ of $L(y)=0$ on $-\infty<x<\infty$ satisfying $\phi(x_0)=\alpha_1,\ \phi'(x_0)=\alpha_2,\ldots,\phi^{(n-1)}(x_0)=\alpha_n$.

22. Let $\phi_1,\ldots,\phi_n$ be n solutions of $L(y)=0$ on an interval I and let x_0 be any point in I. Prove that

$$W(\phi_1,\ldots,\phi_n)\ (x)=\exp\left[-\int_{x_0}^{x} a_1(t)dt\right]W(\phi_1,\ldots,\phi_n)(x_0)$$

23. Find the first kind solution of $x^2y''+xy'+(x^2-\alpha^2)y=0$.

24. Let M,N be two real valued functions which have continuous first partial derivatives on some rectangle $R:|x-x_0|\le a,\ |y-y_0|\le b$. Prove that the equation $M(x,y)+N(x,y)y'=0$ is exact in R if and only if $\dfrac{\partial M}{\partial y}=\dfrac{\partial N}{\partial x}$ in R.
