MAY 2025 **P/ID 37503/PMA1C**

Time : Three hours Maximum : 80 marks

PART A — (10 × 2 = 20 marks)

Answer any TEN questions.

1. Find all the solutions of the equation $y'' - 4y = 0$.

2. State Uniqueness theorem for linear equations with constant coefficients.

3. Define Wronskian.

4. Check whether the sets of functions $\phi_1(x) = 1$, $\phi_2(x) = x$, $\phi_3(x) = x^2$ defined on $-\infty < x < \infty$ are linearly independent or dependent.

5. State Uniqueness theorem for linear equations with variable coefficients.

6. Define linear space of functions.

7. Compute the indicial polynomial and its roots of

$$4x^2y'' + (4x^4 - 5x)y' + (x^2 + 2)y = 0$$

8. Define Bessel function of order α of the first kind.

9. Write the Lipschitz condition.

10. Solve $2xydx + (x^2 + 3y^2)\,dy = 0$.

11. Define regular singular point.

12. Write the Legendre equation.

PART B — (5 × 6 = 30 marks)

Answer any FIVE questions.

13. Let ϕ_1, ϕ_2 are two solutions of $L(y) = 0$ on an interval I containing a point x_0. Prove that

$$W(\phi_1, \phi_2)(x) = e^{-a_1(x-x_0)} W(\phi_1, \phi_2)(x_0)$$

14. Find all the solutions of $y'' + 4y = \cos x$.

15. Find all real valued solutions of the equation $y^{(4)} - 5y'' + 4y = 0$.

16. One solution of $x^2y''' - 3x^2y'' + 6xy' - 6y = 0$ for $x > 0$ is $\phi_1(x) = x$. Find a basis for the solutions for $x > 0$.

17. Compute the indicial polynomials and their roots for the equation

$$x^2y'' + (x + x^2)y' - y = 0$$

18. Suppose the equation $M(x, y) + N(x, y)\,y' = 0$ is exact in a rectangle R, and F is a real valued function such that

$$\frac{\partial F}{\partial x} = M,\ \frac{\partial F}{\partial y} = N$$

in R. Prove that every differential function ϕ defined implicity by a relation $F(x, y) = c$, (c = constant) is a solution and every solution whose graph lies in R arises this way.

19. Show that the equation $f(x, y) = x^3 e^{-xy^2}$, on $S: 0 \leqq x \leqq a$, $|y| < \infty, (a > 0)$ satisfy Lipschitz conditions.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

20. Let ϕ be any solution of $L(y) = y'' + a_1y' + a_2y = 0$ on an interval I containing a point x_0. Prove that for all x in I,

$$\|\phi(x_0)\| e^{-k|x-x_0|} \leqq \|\phi(x)\| \leqq \|\phi(x_0)\| e^{k|x-x_0|}$$

where $\|\phi(x)\| = \left[|\phi(x)|^2 + |\phi'(x)|^2\right]^{\frac{1}{2}}$, $k = 1 + |a_1| + |a_2|$

21. Find a particular solution of the equation $y'' + y = xe^x \cos 2x$ by using annihilator method.

22. State and prove Existence theorem for analytic coefficients.

23. Show that $x^{1/2} J_{1/2}(x) = \dfrac{\sqrt{2}}{\Gamma\left(\dfrac{1}{2}\right)} \sin x$.

24. Prove that the successive approximation ϕ_k, defined by $\phi_0(x)\, y_0$

$$\phi_{k+1}(x) = y_0 + \int_{x_0}^{x} f(t, \phi_k(t))\, dt, \ (k = 0,1,2,\ldots),$$

exist as continuous functions on $I : |x - x_0| \leqq \alpha = \min\{a, b/M\}$ and $\{x, \phi_k(x)\}$ is in R for x in I. Also, prove that ϕ_k satisfy

$|\phi_k(x) - y_0| \leqq M|x - x_0|$ for all x in I.
