DECEMBER 2022 **P/ID 37504/PMA1D**

Time : Three hours Maximum : 80 marks

SECTION A — (10 × 2 = 20 marks)

Answer any TEN questions.

1. Define complete Bipartite Graph.

2. Define a simple graph.

3. Define Euler tour.

4. Define a cut edge.

5. State Menger's theorem.

6. Define Hamilton cycle.

7. State Vizing's theorem.

8. Define covering of a G.

9. Define k-chromatic number.

10. Define edge covering number.

11. Write Jordan curve theorem.

12. Define a planar graph.

SECTION B — (5 × 6 = 30 marks)

Answer any FIVE questions.

13. Prove that an edge e of G is a cut edge of G if and only if e is contained in no cycle of G.

14. State and prove Chvátal Theorem.

15. Prove that a connected graph has an Eulerian trial if and only if it has atmost two vertices of odd degree.

16. Let M be a matching and K be a covering such that $|M| = |K|$. Prove that M is a maximum matching and K is a minimum covering.

17. For any two integers $k \geq 2$ and $l \geq 2$, prove that $r(k,l) \leq r(k,l-1) + r(k-1,l)$.

18. With the usual notations, prove that $r(k,l) \leq \binom{k+l-2}{k-1}$.

19. Prove that K_5 is nonplanar.

SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

20. Prove that a graph is bipartite *if* and only if it contains no odd cycle.

21. Prove that a graph G with $v \geq 3$ is 2-connected if and only if any two vertices of G are connected by atleast two internally-disjoint paths.

22. Prove that a matching M *in* G is a maximum matching if and only if G contains no M-augmenting path.

23. Prove that $r(k,k) \geq 2^{k/2}$.

24. State and prove the five-color Theorem.

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