DECEMBER 2023 **P/ID 37504/PMA1D**

Time : Three hours Maximum : 80 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Prove that $\delta \leq 2\varepsilon / v \leq \Delta$ with usual notations.

2. Show that if G is simple and $\delta \geq k$, then G has a path of Length k.

3. Show that if G is a forest, then $\varepsilon = v - \omega$.

4. Find a simple graph G with $k < k' < \delta$.

5. If G is hamiltonian, then for every nonempty proper subset S of V, show that $\omega(G - S) \leq |S|$.

6. State a necessary and sufficient condition for a graph to have a perfect matching.

7. Prove that $X' \geq \Delta$.

8. Show that a set $S \subseteq V$ is an independent set of G if and only if $V \setminus S$ is a covering of G.

9. Prove that $r(2, 2) = 2$.

10. Show that, for any graph G, $X \leq \Delta + 1$.

11. If G is a simple planar graph, prove that $\delta \leq 5$.

12. Prove that $K_{3,3}$ is non-planar.

PART B — ($5 \times 6 = 30$ marks)

Answer any FIVE questions.

13. Show that if $\delta \geq 2$, then G contains a cycle.

14. If G is a tree, prove that $\varepsilon = v - 1$.

15. If G is a simple graph with $v \geq 3$ and $\delta \geq v/2$, prove that G is hamiltonian.

16. If G is a k-regular bipartite graph with $k > 0$, prove that G has a perfect matching.

17. Show that $X'(K_{m,n}) = \Delta(K_{m,n})$.

18. For any two integers $k \geq 2$ and $l \geq 2$, prove that $r(k, l) \leq r(k, l-1) + r(k-1, l)$.

19. State and prove Euler's formula.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

20. Show that a graph is bipartite if and only if it contains no odd cycle.

21. Let G be a simple graph with degree sequence $(d_1, d_2, ..., d_v)$, where $d_1 \le d_2 \le ... \le d_v$ and $v \ge 3$. Suppose that there is no value of m less than $v/2$ for which $d_m \le m$ and $d_{V-m} < v - m$. Prove that G is hamiltonian.

22. Show that a matching M in G is a maximum matching if and only if G contains no M-augmenting path.

23. State and prove Dirac's theorem on critical graphs.

24. Show that G is a connected planar graph with $k \ge 3$, then $\varepsilon \le k(v-2)/(k-2)$. Hence deduce that the Petersen graph is non-planar.

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