DECEMBER 2024 **P/ID 37504/PMA1D**

Time : Three hours Maximum : 80 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Define graph.

2. Define walk and trail of graph G.

3. Prove that in any graph, the number of vertices of odd degree is even.

4. Define complete graph.

5. Define Euler trail and Euler tour of G.

6. Define matching in G.

7. Define k-edge-colouring.

8. Define (k, l)-Ramsey graph.

9. Define chromatic number.

10. Define Jordan curve.

11. Define faces of G.

12. Define Tait colouring.

PART B — ($5 \times 6 = 30$ marks)

Answer any FIVE questions.

13. Prove that if G is a tree, then $\varepsilon = v - 1$.

14. Prove that G is hamiltonian, if G is a simple graph with $v \geq 3$ and $\delta \geq v/2$.

15. Suppose G is a connected graph that is not an odd cycle. Prove that G has a 2-edge colouring in which both colours are represented at each vertex of degree at least two.

16. Let $(E_1, E_2, ..., E_k)$ be an optimal k-edge colouring of G. If there is a vertex u in G and colours i and j such that i is not represented at u and j is represented at least twice at u, prove that the component of $G[E_i \cup E_j]$ that contains u is an odd cycle.

17. Prove that $r(k, k) \geq 2^{k/2}$.

18. Prove that $\pi_k(G) = \pi_k(G - e) - \pi_k(G \cdot e)$ for any edge e of G, if G is simple.

19. Prove that $v - \varepsilon + \phi = 2$ if G is a connected plane graph.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

20. Prove that an edge e of G is a cut edge of G if and only if e is contained in no cycle of G.

21. Prove that $k \leq k' \leq \delta$.

22. Prove that if G is simple, prove that either $\chi' = \Delta$ or $\chi' = \Delta + 1$.

23. Prove that $\chi \leq \Delta$ if G is a connected simple graph and is neither an odd cycle nor a complete graph.

24. State and prove the five-color theorem.

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