MAY 2022 **P/ID 37504/PMA1D**

Time : Three hours Maximum : 80 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Define incidence matrix of G.

2. Define complete Graph.

3. State Menger's theorem.

4. Define acyclic graph.

5. Define eulerian graph.

6. Define Hamilton cycle.

7. Define edge chromatic number.

8. Define a matching in G.

9. State Brook's theorem.

10. State Ramsey theorem.

11. Write Jordna curve theorem.

12. Define a planar graph.

PART B — ($5 \times 6 = 30$ marks)

Answer any FIVE questions.

13. Let T be a spanning tree of a connected graph G and let e be any edge of T. Prove that

    (a) the cotree $\overline{T}$ contains no bond of G;

    (b) $\overline{T} + e$ contains a unique bond of G.

14. If G is a simple graph with $v \geq 3$ and $\delta \geq v/2$, prove that G is hamiltonian.

15. Prove that $C(G)$ is well defined.

16. Let G be a connected graph that is not an odd cycle. Prove that G has a 2-edge colouring in which both colours are represented at each vertex of degree atleast two.

17. If $\delta > 0$, prove that $\alpha' + \beta' = v$.

18. In a critical graph, prove that no vertex is a clique.

19. In a connected plane graph, prove that $v - \varepsilon + \phi = 2$.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

20. Prove that a graph is bipartite if and only if it contains no odd cycle.

21. Prove that a nonempty connected graph is eulerian if and only if it has no vertices of odd degree.

22. State and prove Vizing's Theorem.

23. If G is a connected simple graph and is neither an odd cycle nor a complete graph, then prove that $\chi \leq \Delta$.

24. State and prove the five-color Theorem.

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