MAY 2023 **P/ID 37504/PMA1D**

Time : Three hours Maximum : 80 marks

PART A — (10 × 2 = 20 marks)

Answer any TEN questions.

1. Show that if G is simple, then $\varepsilon \leq \binom{\upsilon}{2}$.

2. Show that in any graph the number of vertices of odd degree is even.

3. Prove that every nontrivial tree has at least two vertices of degree one.

4. Draw a graph with $\kappa < \kappa' < \delta$.

5. What is the connectivity of $K_{m,n}$?

6. What are the blocks of a tree?

7. When does the graph K_n has a perfect matching?

8. State marriage theorem.

9. Show that $\alpha + \beta = \gamma$.

10. Prove that $r(3\ 3) = 6$.

11. Define K-edge colouring.

12. Prove that K_5 is nonplanar.

PART B — (5 × 6 = 30 marks)

Answer any FIVE questions.

13. Show that if a k-regular bipartite graph with $k > 0$ has bipartition (X,Y), then $|X| = |Y|$.

14. Show that an edge e of G is a cut edge of G if and only if e is contained in no cycle of G.

15. Show that a connected graph has an Euler trial if and only if it has at most two vertices of odd degree.

16. If G is a k-regular bipartite graph with $k > 0$. prove that G has a perfect matching.

17. Show that the Petersen graph is 4-edge-chromatic.

18. If $\delta > 0$, prove that $\alpha' | \beta' = \upsilon$.

19. State and prove Euler's formula.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

20. Show that a graph is bipartite if and only if it contains no odd cycle.

21. Prove that $K \leq K' \leq \delta$.

22. State and prove Hall's theorem.

23. If G is a connected simple graph and is neither an odd cycle nor a complete graph, prove that $\chi \leq \Delta$.

24. Show that every planar graph is 5-vertex-colourable.

———————