MAY 2024 **P/ID 37504/PMA1D**

Time : Three hours Maximum : 80 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Show that in any graph, the sum of degrees of the vertices is an even number.

2. When do we say two graphs are isomorphic?

3. If G is a tree, prove that every edge is a cut edge.

4. Find K and K' for the complete graph K_n.

5. For what values of m,n the complete bipartite graph $K_{m,n}$ is Eulerian?

6. For what values of m,n the complete bipartite graph $K_{m,n}$ is Hamiltonian?

7. Define perfect matching.

8. Prove with the usual notation that $\chi'(K_{m,m}) = m$.

9. Prove that $\gamma(3,4) = 9$.

10. Show that every critical graph is a block.

11. Show that all planar embeddings of a given connected planner graph have the same number of faces.

12. If G is a simple planar graph with $\gamma \geq 3$, prove that $\varepsilon \leq 3v - 6$.

PART B — ($5 \times 6 = 30$ marks)

Answer any FIVE questions.

13. Show that if G is simple and $\varepsilon > \binom{v-1}{2}$, then G is connected.

14. Show that a vertex v of a tree G is a cut vertex of G if and only if $d(v) > 1$.

15. Show that a connected graph has an Euler trial if and only if it has at most two vertices of odd degree.

16. Prove that $C(G)$ is well defined.

17. If G is a R-regular bipartite graph with $k > 0$, prove that G has a perfect matching.

18. Show that $r(k,l) \leq \binom{k+l-2}{k-1}$.

19. With usual notation, if G is a connected planar graph, prove that $v - \varepsilon + \phi = 2$.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

20. Let G be a graph with $v-1$ edges. Show that the following three statements are equivalent :

    (a) G is connected ;

    (b) G is acyclic ;

    (c) G is a tree.

21. Show that a graph G with $v \geq 3$ is 2-connected if and only if any two vertices of G are connected by at least two internally-disjoint paths.

22. If G is simple, prove that either $\chi' = \Delta$ or $\chi' = \Delta + 1$.

23. State and prove Brook's theorem.

24. Show that a graph is planar if and only if its block is planar. Hence deduce that a minimal non-planar graph is a simple block.

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