MAY 2025 **P/ID 37504/PMA1D**

Time : Three hours Maximum : 80 marks

PART A — (10 × 2 = 20 marks)

Answer any TEN questions.

1. Define incidence matrix of G.

2. Define induced subgraph of G.

3. Prove that $\varepsilon \geq v - 1$ if G is connected.

4. Define connectivity of G.

5. State Menger's theorem.

6. Define M-alternating path in G.

7. Define edge chromatic number,

8. Define clique of a simple graph G.

9. Define critical graph.

10. Define planar embedding of G.

11. Define degree of a face f in G.

12. Define k-face colorings.

PART B — (5 × 6 = 30 marks)

Answer any FIVE questions.

13. Prove that an edge e of G is a cut edge of G if and only if e is contained in no cycle of G.

14. Prove that a graph G with $v > 3$ is 2-connected if and only if any two vertices of G are connected by at least two internally-disjoint paths.

15. Prove that the closure of G is well defined.

16. Prove that $\chi' = \Delta$ if G is bipartite.

17. Prove that $\alpha' + \beta' = v$, if $\delta > 0$.

18. Prove that $\delta \geq k - 1$ if G is k-critical.

19. Prove that $\varepsilon \leq 3v - 6$ if G is a simple planar graph with $v \geq 3$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

20. Prove that a graph is bipartite if and only if it contains no odd cycle

21. Prove that a nonempty connected graph is Eulerian if and only if it has no vertices of odd degree.

22. Let G be a bipartite graph with bipartition (X,Y). Prove that G contains a matching that saturates every vertex in X if and only is $|N(S)| \geq |S|$ for all $S \subseteq X$.

23. State and prove Dirac's theorem.

24. Prove that every planar graph is 5-vertex-colourable.

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