DECEMBER 2023 **P/ID 37505/PMA1E**

Time : Three hours Maximum : 80 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Write the properties of relation R.
2. Define an upper bound.
3. Write the duality principle.
4. Define a switching circuit.
5. What is an NOT – gate?
6. What is a NAND – gate?
7. What is a cyclotomic polynomial?
8. Define a Frobenius automorphism.
9. Define a primitive polynomial.
10. Write a decomposable polynomial.
11. What is the canonical basic matrix?
12. Define an orthogonal code.

PART B — ($5 \times 6 = 30$ marks)

Answer any FIVE questions.

13. State and prove De Morgan's laws.

14. Let B be a Boolean algebra and let I be nonempty subset of B. Prove that the following conditions are equivalent: (a) $I \leq B$ (b) For all i, $j \in I$ and $b \in B$: $i + j \in I$ and $b \leq i \Rightarrow b \in I$; (c) I is the kernal of a Boolean homomorphism from B into another Boolean algebra.

15. Determine the switching circuit for $p = x_1(x'_2(x_6 + x_3(x_4 + x'_5)) + x_7(x_3 + x_6)x'_8)$.

16. Let n be a positive integer and let F be a field whose characteristic does not divide n. (a) Prove that there is a finite extension K of F which contains a primitive nth root of unity. (b) If α is a primitive nth root of unity, then prove that $F(\alpha)$ is the splitting field of $f = x^n - 1$ over F.

17. Prove that $x^{q^n} - x = \Pi_i f_i$, where the product is extended over all distinct monic irreducible polynomials over F_q with degree a divisor of n.

18. Prove that the number of monic irreducible polynomials of degree k over F_q is given by $I_q(k) = \frac{1}{k}\sum_{d/k} \mu\left(\frac{k}{d}\right) q^d$.

19. Prove that a linear code $C \subseteq V_n$ is cyclic if and only if C is an ideal in V_n.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

20. Let B be a finite Boolean algebra and let A denote the set of all atoms in B. Prove that B is isomorphic to $P(A)$, $(B,\wedge,V) \cong_b (P(A),\cap,\cup)$.

21. Explain the addition of two binary numbers using half adders.

22. Let F be a finite field of characteristic p. Prove that F contains p^n elements, where $n = [F : Z_p]$.

23. State and prove Chinese Remainder theorem for polynomials.

24. State and prove Gilbert Varshamov Bound theorem.

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