DECEMBER 2024 **P/ID 37505/PMA1E**

Time : Three hours Maximum : 80 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Define partial order relation.

2. Define system of normal forms.

3. Draw the sub junction gate.

4. Give the Minimal forms of the polynomials p_2, p_3, p_5 and p_7.

5. Define bridge circuit.

6. Define primitive root modulo p.

7. Define n-th cyclotomic polynomial over a field F.

8. Show that the three irreducible polynomials of degree 4 over $\mathbb{F}_2$ are x^4+x+1, x^4+x^3+1, $x^4+x^3+x^2+x+1$.

9. Define cyclotomic coset modulo n containing s.

10. Prove that $x^4 - x^2 - 1$ is irreducible over $\mathbb{F}_3$.

11. Define encoding matrix.

12. Define Hamming weight of a vector.

PART B — ($5 \times 6 = 30$ marks)

Answer any FIVE questions.

13. Prove that a lattice L is modular if and only if none of its sublattices is isomorphic to the pentagon lattice.

14. Let V be a lattice. Prove the following implications:

    (a) (V is a Boolean algebra) $\Rightarrow$ (V is relatively complemented).

    (b) (V is finite and sectionally complemented) $\Rightarrow$ (every $0 \neq a \in V$ is a join of finitely many atoms).

15. Simplify the following using Karnaugh diagrams.

    (a) $x_1x_3 + x_1'x_3x_4 + x_2x_3'x_4 + x_2'x_3x_4$

    (b) $(x_1' + x_2)(x_3 + x_4')(x_2 + x_4'x_1)$.

16. Prove the following:

(a) $x^n - 1 = \prod_{d|n} Q_d$.

(b) $Q_{p^m} = 1 + x^{p^{m-1}} + \cdots + x^{(p-1)p^{m-1}}$.

17. Prove that there are exactly $\phi(e)/m$ irreducible polynomials of degree m and order e over $\mathbb{F}_q$.

18. Prove the following:

(a) $\sum_{a \in \mathbb{F}_q} \chi(a) = 0$ for a nontrivial character of $\mathbb{F}_q$.

(b) The characters of $\mathbb{F}_q$ form a group which is isomorphic to the group $(\mathbb{F}_q, +)$.

19. Prove that a linear code $C \subset V_n$ is cyclic if and only if C is a principal ideal in V_n, generated by $g \in C$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

20. Determine the minimal form of p, which is given in disjunctive normal form $d = v'w'x'y'z' + v'w'x'yz' + v'w'xy'z' + v'w'xyz' + v'wx'yz' + v'wxy'z' + v'wxyz' + vw'x'y'z' + vw'x'y'z + vw'xy'z + vwx'yz' + vwxy'z' + vwxyz' + cwxyz$.

21. Solve the system of equations $x_1x_3 = x_2 + x_1x_2x_3'$, $x_1' + x_3 = x_1x_2$.

22. Find the cyclotomic polynomials Q_{36} and Q_{105}.

23. Prove that a polynomial $f \in \mathbb{F}_q[x]$ of degree k is primitive over $\mathbb{F}_q$ if and only if f is monic, $f(0) \neq 0$ and the order of f is equal to $q^m - 1$.

24. State arid prove MacWilliams Identity.

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