MAY 2023 **P/ID 37505/PMA1E**

Time : Three hours Maximum : 80 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Define a poset.
2. Define an inverse relation.
3. Define an algebraic lattice.
4. Define a switch.
5. What is an AND - gate?
6. What is a NOR - gate?
7. Define n-pole circuit.
8. Define an associated cyclotomic field.
9. Define cyclotomic coset.
10. Write an exponent function
11. Write is the parity check matrix?
12. Define a (n, k, d) code.

PART B — ($5 \times 6 = 30$ marks)

Answer any FIVE questions.

13. Let B be a boolean algebra. Prove that the set $P_n(B)$ is a Boolean algebra and a subalgebra of the Boolean algebra $F_n(B)$ of all functions from B_n into B.

14. Let $p, q \in P_n$; $p \sim q$ and let B be an arbitrary Boolean algebra. Prove that $\overline{p}_B = \overline{q}_B$.

15. Determine the symbolic representation of the circuit given by $p = ((x_1 x_2)(x_3 + x_4)(x_1 x_2 + x_3' x_4)$.

16. Let p be a prime number, F a field not of characteristic p and m a Positive integer. Prove that $Q_{p^m} = 1 + x^{p^{m-1}} + ... + x^{(p-1)p^{m-1}}$.

17. Let $f \in F_q(x)$ be an irreducible polynomial over F_q of degree $m \geq 2$. Prove that ord *f* is equal to the order of any root of *f* in $F_{q^m}^*$.

18. Let F be a field. Prove that every $f \in F(x)$ can be written as a decomposition of finitely many indecomposable polynomials

19. Let **H** be a parity check matrix of an (n, k, d) code C with $n > k$. Prove that the following hold:

    (i) dim $C = k = n - Rk$ **H**

    (ii) $d = mld$ **H**,

    (iii) $d \leq n - k + 1$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

20. (i) Let $(L, \leq)$ be a lattice ordered set. If we define $x \wedge y := \inf(x, y), x \vee y : \sup(x, y)$, prove that $(L, \wedge, \vee)$ is an algebraic lattice.

    (ii) Let $(L, \wedge, \vee)$ is an algebraic lattice. If we define $x \leq y; \Leftrightarrow x \wedge y = x$, prove that $(L, \leq)$ is a lattice ordered set.

21. Explain the addition of three one digit binary numbers using full adders.

22. State and prove Mobius inversion Formula

23. Prove that a polynomial $f \in F_q(x)$ of degree m is primitive if and only if F is monic, $f(0) \neq 0$ and the order of f is equal to $q^m - 1$.

24. Let h be a check polynomial of a cyclic code $C \subseteq V_n$ with generator polynomial g and $v \in V_n$. Prove that $u \in C \Leftrightarrow v * h = 0$

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