MAY 2024 **P/ID 37505/PMA1E**

Time : Three hours Maximum : 80 marks

PART A — (10 × 2 = 20 marks)

Answer any TEN questions.

1. Define a linearly ordered set.

2. Define a lower bound of a set.

3. Define a sublattice.

4. Define a complementation switch.

5. What is an identity - gate?

6. What is a NOR - gate?

7. What is a Mobius function?

8. Define a root of unity.

9. Define a decomposable polynomial.

10. Define an exponent.

11. What is the equivalent of codes?

12. Define a syndrome.

PART B — (5 × 6 = 30 marks)

Answer any FIVE questions.

13. Prove that the elements of an arbitrary lattice satisfy the following distributive inequalities :
$x \wedge (y \vee z) \geq (x \wedge y) \vee (x \wedge z)$ and
$x \vee (y \wedge z) \leq (x \vee y) \wedge (x \vee z)$.

14. Prove that a polynomial $p \in P_n$ is equivalent to the sum of all prime implicants of p.

15. Determine the switching circuit for
$p = x_1 (x_2(x_3 + x_4) + x_3(x_5 + x_6))$.

16. Let p be a prime and let m, n be natural numbers :
    (a) If F_{p^m} is a subfield of F_{p^n} then prove that m/n.
    (b) If m/n then prove that $F_{p^m} \to F_{p^n}$.

17. Let $f \in F_q[x]$ be a polynomial of degree $m \geq 1$ with $f(0) \neq 0$. Prove that there exist a positive integer $e \leq q^m - 1$ such that f divides $x^e - 1$.

18. Prove that an irreducible polynomial f over F_q of degree k divides $x^{q^n} - x$ if and only if k divides n.

19. Prove that the following :

(a) $S(y) = 0 \Leftrightarrow y \in C$

(b) $S(y) = S(w) \Leftrightarrow y + C = w + C \Leftrightarrow y$ and w are in the same coset.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

20. Let B be a Boolean Algebra. Prove that the set $P_n(B)$ is a Boolean algebra and a subalgebra of the Boolean algebra $F_n(B)$ of all functions from B_n into B.

21. A motor is supplied by three generators. The operation of each generator is monitored by a corresponding switching element which closes a circuit as soon as a generator fails. We demand the following conditions from the electrical monitoring system :

(a) A warning lamp lights up if one or two generators fail.

(b) An acoustic alarm is initiated if two or all three generators fail.

Determine a symbolic representation as a mathematical model of this problem.

22. Prove that the following :

(a) Any finite field F is of order p^n where $p = char\ F$ is a prime and n is a positive integer.

(b) For any prime p and any $n \in N$ there is a field of order p^n.

(c) Any field of order p^n is the splitting field of $x^{p^n} - x$ and also of $x^{p^{n-1}} - 1 \in Z_p[x]$.

23. Prove that the following :

(a) $f = \prod_{s \in F_q} \gcd(f, h - s)$.

(b) The number of monic distinct irreducible factors f_i of f is equal to the dimension of the null space of the matrix $Q - I$.

24. Let G be a generator matrix of a linear code C. Prove that the rows of G from a basis of C.