DECEMBER 2022 **P/ID 37506/PMA2F**

Time : Three hours Maximum : 80 marks

SECTION A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Define algebraic of degree n.

2. Define algebraic number.

3. Define root of a polynomial.

4. Define simple extension.

5. Define fixed field of G.

6. Define Galois group of polynomials.

7. Prove that the multiplicative group of non zero elements of a finite field is cyclic.

8. Prove that any two finite fields having the same number of elements are isomorphic.

9. Define solvable group.

10. When a division algebra D is said to be algebraic over a field F.

11. Define automorphism in a field K.

12. Define normal extension.

SECTION B — ($5 \times 6 = 30$ marks)

Answer any FIVE questions.

13. If L is an algebraic extension of K and if K is an algebraic extension of F, then prove that L is an algebraic extension of F.

14. Prove that $(f(x)g(x))' = f'(x)g(x) + f(x)g'(x)$, for any $f(x),\ g(x) \in F[x]$.

15. Prove that K is s normal extension of F then prove that K is the splitting field of some polynomial over F.

16. Let F be a finite field with q elements and suppose that $F \subset K$, where K is also a finite field. Prove that K has q^n elements when $n = [K : F]$.

17. Let C be the field of complex numbers and suppose that the division ring D is algebraic over C. Prove that $D = C$.

18. If L is a finite extension of F and if K is a subfield of L which contains F, then prove that $[K : F]/[L : F]$.

19. Prove that for every prime number p and every positive integer m there exists a field having p^m elements.

SECTION C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

20. Prove that the number e is transcendental.

21. Prove that a polynomial of degree n over a field can have at most n roots in any extension field.

22. If K is a field and if $\sigma_1,\ldots,\sigma_n$ are distinct automorphisms of K, then prove that it is possible to find $a_1,\ldots,a_n$ not all 0, in K such that $a_1\sigma_1(u)+a_2\sigma_2(u)+\cdots+a_n\sigma_n(u)=0$ for all $u\in K$.

23. State and prove Wedderburn's theorem.

24. State and prove Frobenius theorem.

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