DECEMBER 2023 **P/ID 37506/PMA2F**

Time : Three hours Maximum : 80 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Define algebraic element.

2. Define algebraic extension.

3. Define root of a polynomial.

4. Define simple extension.

5. Define Galois group.

6. Define fixed field of G.

7. Define finite field.

8. Let F be a finite field, then prove that F has p^m elements, where the prime number p is the characteristic of F.

9. Define solvable group.

10. When a division algebra D is said to be algebraic over a field F.

11. Define splitting field.

12. Define algebraic of degree n.

PART B — ($5 \times 6 = 30$ marks)

Answer any FIVE questions.

13. If L is an algebraic extension of K and if K is an algebraic extension of F, then prove that L is an algebraic extension of F.

14. State and prove Remainder theorem.

15. Prove that the fixed field of G is a subfield of K.

16. Let $f(x) \in F[x]$ be of degree $n \geq 1$, prove that there is an extension E of F of degree at most $n!$ in which $f(x)$ has n roots.

17. If K is a finite extension of F, then prove that $G(K, F)$ is a finite group and its order, $o(G(K, F))$ satisfies $o(G(K, F)) \leq [K : F]$.

18. Prove that for every prime number p and every positive integer m there exists a field having p^m elements.

19. Prove that the general polynomial of degree $n \geq 5$ is not solvable by radicals.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

20. If L is a finite extension of K and if K is a finite extension of F, then prove that L is a finite extension of F and $[L : F] = [L : K][K : F]$.

21. If F is of characteristic 0 and if a, b are algebraic over F, then there exist an element $c \in F(a, b)$ such that $F(a, b) = F(c)$.

22. State and prove the fundamental theorem of Galois theory.

23. State and prove Wedderburn's theorem.

24. State and prove Frobenius theorem.

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