MAY 2022 **P/ID 37506/PMA2F**

Time : Three hours Maximum : 80 marks

PART A — (10 × 2 = 20 marks)

Answer any TEN questions.

1. Define an algebraic element.

2. Define an algebraic number.

3. Find an extension field over Q, such that $x^2 - 2$ has roots.

4. Define a simple extension.

5. Define the fixed field of *G*.

6. Define normal extension.

7. Prove that the multiplicative group of non zero elements of a finite field is cyclic.

8. If the finite field F has p^m elements then prove that every $a \in F$ satisfies $a^{p^m} = a$.

9. Define a solvable group.

10. When do we say, a division algebra D is said to be algebraic over a field F.

11. Define the degree of K over F for $K \supset F$ - extension of fields.

12. Define a automorphism in a field K.

PART B — ($5 \times 6 = 30$ marks)

Answer any FIVE questions.

13. If L is an algebraic extension of K and if K is an algebraic extension of F, then prove that L is an algebraic extension of F.

14. State and prove Remainder theorem.

15. Prove that the fixed field of G is a subfield of K.

16. If F is a finite field and $\alpha \neq 0$, $\beta \neq 0$ are two elements of F then prove that we can find elements a and b in F such that $1 + \alpha a^2 + \beta b^2 = 0$.

17. Prove that, any general polynomial of degree $n \geq 5$ is not solvable by radicals.

18. Prove that there is an isomorphism τ^{**} of $F[x]/(f(x))$ onto $F'[t]/(f'(t))$ with the property that for every $\alpha \in F$, $\alpha\tau^{**} = \alpha'$, $(x + (f(x)))\tau^{**} = t + (f'(t))$.

19. Let K be the splitting field of $f(x)$ in $F[x]$ and let $p(x)$ be an irreducible factor of $f(x)$ in $F[x]$. If the roots of $p(x)$ are $\alpha_1, \alpha_1, \cdots, \alpha_r$, then prove that for each i, there exists an automorphism σ_i in $G(K, F)$ such that $\sigma_i(\alpha_1) = \alpha_i$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

20. Prove that the element $a \in K$ is algebraic over F if and only if $F(a)$ is a finite extension of F.

21. If $p(x)$ is a polynomial in $F[x]$ of degree $n \geq 1$ and its irreducible over F, then prove that there is an extension E of F, such that $[E : F] = n$, in which $p(x)$ has a root.

22. If K is a field and if $\sigma_1, \cdots, \sigma_n$ are distinct automorphisms of K, then prove that it is possible to find $a_1, \cdots, a_n$ not all 0, in K such that $a_1\sigma_1(u) + a_2\sigma_2(u) + \cdots + a_n\sigma_n(u) = 0$ for all $u \in K$.

23. State and prove Wedderburn's theorem.

24. State and prove Frobenius theorem.

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