DECEMBER 2022 **P/ID 37507/PMA2G**

Time : Three hours Maximum : 80 marks

SECTION A — ($10 \times 2 = 20$ marks)

Answer any TEN questions

1. Define Lebesgue outer measure.
2. Define essential supremum.
3. Define essential infimum.
4. State Lebesgue's dominated convergence theorem.
5. State Fatou's Lemma.
6. Write Bessel's inequality.
7. State Jordan's test.
8. State Weierstrass Approximation Theorem.
9. Write Taylor's formula.
10. State Dini's theorem.
11. Define saddle point.
12. Define open mapping.

SECTION B — ($5 \times 6 = 30$ marks)

Answer any FIVE questions

13. Let $\{E_i\}$ be sequence of measurable set. Prove the following:

    (a) If $E_1 \subseteq E_2 \subseteq \ldots$ then $m(\lim E_i) = \lim m(E_i)$

    (b) If $E_1 \supseteq E_2 \supseteq \ldots$ and $m(E_i) < \infty$ for each i, then $m(\lim E_i) = \lim m(E_i)$

14. Prove that not every measurable set is a Borel set.

15. Let f and g be non-negative measurable functions. Then prove that

$$\int f\,dx + \int g\,dx = \int (f+g)\,dx$$

16. State and prove Riesz-Fischer Theorem

17. State and prove that chain rule for derivatives.

18. State and prove the Mean-value Theorem.

19. A quadric surface with center at the origin has the equation.

$$Ax^2 + By^2 + Cz^2 + 2Dyz + 2Ezx + 2Fxy = 1$$

Find the lengths of its semi-axes.

SECTION C — ($3 \times 10 = 30$ marks)

Answer any THREE questions

20. Prove that the outer measure of an interval equals its length.

21. Let f be bounded and measurable on a finite interval $[a,b]$ and $\in > 0$ Then prove that there exist

    (a) a step function h such that $\int_a^b |f-h|dx < \in$,

    (b) a continuous function g such that g vanishes outside a finite interval and $\int_a^b |f-g|dx < \in$.

22. State and prove Fejer's theorem.

23. If both partial derivatives $D_r f$ and $D_k f$ exist in an n-ball $B(c;\delta)$ and if both are differentiable at c, then prove that $D_{r,k}f(c) = D_{k,r}f(c)$.

24. State and prove Implicit function theorem.

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