DECEMBER 2023 **P/ID 37507/PMA2G**

Time : Three hours Maximum : 80 marks

PART A — (10 × 2 = 20 marks)

Answer any TEN the questions

1. Define the Lebesgue outer measure.

2. Define a Lebesgue measurable set.

3. Define the integral of a measurable function.

4. State Lebesgue's Monotone Convergence Theorem.

5. Define the Fourier series and Fourier coefficients of f relative to S.

6. Write Dirichlet's kernel.

7. State the Weierstrass Approximation Theorem.

8. Write Dini's test.

9. Write the first-order Taylor's formula.

10. Write the Chain rule.

11. Define stationary point and saddle point.

12. Define an open mapping.

PART B — ($5 \times 6 = 30$ marks)

Answer any FIVE questions

13. Prove that not every measurable set is a Borel set.

14. Let f and g be non-negative measurable functions. Prove that

$$\int f dx + \int g dx = \int (f + g) dx$$

15. State and prove Lebesgue's dominated convergence theorem.

16. State and prove Fejérs theorem.

17. State and prove the Mean Value Theorem.

18. Let f : $\mathbb{R}^2 \to \mathbb{R}^3$ be defined by $f(x, y) = (\sin x \cos y, \sin x \sin y \cos x \cos y)$.
Determine the Jacobian matrix $Df(x, y)$.

19. State and prove the Second-derivative test for extrema.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions

20. If $m^*(E) < \infty$ then prove that E is measurable if and only if $\forall \in > 0, \exists$ disjoint finite intervals $I_1, \cdots, I_n$ such that $m^*\left(E \Delta \bigcup_{i=1}^{n} I_i\right) < \in$.

21. If f is Riemann integrable and bounded over the finite interval $[a,b]$, then prove that f is integrable and R $\int_a^b f dx = \int_a^b f dx$.

22. State and prove the Riesz-Fischer Theorem.

23. State and prove the Taylor's formula.

24. State and prove Implicit function theorem.

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