DECEMBER 2022 **P/ID 37508/PMA2H**

Time : Three hours Maximum : 80 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. From the partial differential equation by eliminating the arbitrary function from $z = xy + f(x^2 + y^2)$.

2. Find the complete integral of $\sqrt{p} + \sqrt{q} = 1$.

3. Find the complete integral of $z = px + qy + \sqrt{1 + p^2 + q^2}$.

4. Write the Laplace equation.

5. Write the Laplace operator.

6. Define boundary condition – I

7. If $f(t)$ is a continuous function. Prove that $\int_{-\infty}^{\infty} \delta(t-a) f(t) dt = f(a)$.

8. Prove that $\delta(at) = \frac{1}{a}\delta(t), a > 0$.

9. Define the wave equation.

10. Write the Robin condition for wave equation.

11. Define the finite sine transform.

12. Write the D'Alemberts solution of wave equation.

PART B — ($5 \times 6 = 30$ marks)

Answer any FIVE questions.

13. Form the partial differential equation by eliminating the arbitrary function from $z = f(xy/z)$.

14. Find the equation of the tangent line to the space circle $x^2 + y^2 + z^2 = 1, x + y + z = 0$.

15. Find the general spherically symmetric solution of the following Helmholtz equation: $(\nabla^2 - k^2)u = 0$.

16. An infinite one dimensional solid defined by $-\infty < x < \infty$ is maintained at zero temperature initially. There is a heat source of strength $g_s(t)$ units, situated at $x = \xi$, which releases constant heat continuously for $t > 0$. Find an expression for the temperature distribution in the solid for $t > 0$.

17. Assuming the surface of the earth to be flat, which is initially at zero temperature and for time $t > 0$, the boundary surface is being subjected to periodic heat flux $g_0 \cos \omega t$. Investigate the penetration of these temperature variations into the earths surface and show that at a depth x the temperature fluctuates and amplitude of a steady temperature is given by $\frac{g_0}{\sqrt{2}} \sqrt{\frac{2\alpha}{\omega}} \exp\left[-\sqrt{(w/2\alpha)}x\right]$.

18. Consider Maxwell's equation of electromagnetic theory given by $\nabla . E = 4\pi\rho, \nabla . H = 0, \quad \nabla \times E = -\frac{1}{c}\frac{\partial H}{\partial t}$, $\nabla \times H = \frac{4\pi i}{c} + \frac{1}{c}\frac{\partial E}{\partial t}$ where E is an electric field, ρ is electric charge density, H is the magnetic field, i is the current density and c is the velocity of light. Show that in the absence of charges, when $\rho = i = 0$, E and H satisfy the wave questions.

19. Determine the Green's function for the Helmholtz equation for the half space $z \geq 0$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

20. Reduce the Tricomi equation $u_{xx} + xu_{yy} = 0, x \neq 0$, for all x, y to a canonical form.

21. Show that the velocity potential for an irrotational flow of an incompressible fluid satisfies the Laplace solution.

22. The ends A and B of a rod, 10 cm in length, are kept at 0^oC and 100° until the steady state conditions prevails. Suddenly the temperature at the end A is increased to 20^oC and the end B is decreased to 60^oC. Find the temperature distribution in the rod at time t.

23. Find the periodic solution of the wave equation in the form $u(x,t) - Ae^{i(kx+wt)}$, where $i = \sqrt{-1}, k = \pm\frac{w}{c}$ and A is constant.

24. Consider the case when R consists of the half plane defined by $x \geq 0, -\infty < y < \infty$ and hence solve $\nabla^2 u = 0$ in the above region subject to the condition $u = f(y)$ on $x = 0$ using the Green's function technique.
