DECEMBER 2023 **P/ID 37508/PMA2H**

Time : Three hours Maximum : 80 marks

PART A — (10 × 2 = 20 marks)

Answer any TEN questions.

1. Find the PDE by eliminating the arbitrary function from $z = xy + f(x^2 + y^2)$.

2. Write the nonlinear one-dimensional Burger equation.

3. Find the adjoint of the differential operator $L(u) = u_{xx} - u_t$.

4. Write the exterior Dirichlet problem for a circle.

5. Write the solution of Laplace equation in cylindrical coordinates.

6. Write the elementary solutions of diffusion equation.

7. Write any two properties of Dirac delta function.

8. Write the Maxwell's equation of electromagnetic theory.

9. Write the D'Alembert's solution of the one dimensional wave equation.

10. Write the periodic solution of one dimensional wave equation in spherical polar coordinates.

11. Write Helmholtz theorem.

12. Write the Poisson's integral formula.

PART B — ($5 \times 6 = 30$ marks)

Answer any FIVE questions.

13. Find the characteristics of the PDE : $p^2 + q^2 = 2$ and determine the integral surface which passes through $x = 0$, $z = y$.

14. Derive the Poisson equation.

15. In a one dimensional infinite solid, $-\infty < x < \infty$, the surface $a < x < b$ is initially maintained at temperature T_0 and at zero temperature everywhere outside the surface. Show that $T(x,t) = \frac{T_0}{2}\left[erf\left(\frac{b-x}{\sqrt{4\alpha t}}\right) - erf\left(\frac{a-x}{\sqrt{4\alpha t}}\right)\right]$ where erf is an error function.

16. Determine the temperature $T(r, t)$ in the infinite cylinder $0 \le r \le a$ when the initial temperature is $T(r,0) = f(r)$, and the surface $r = a$ is maintained at 0° temperature.

17. Find the D'Alembert's solution of the one-dimensional wave equation.

18. Consider the case when $\mathbb{R}$ consists of the half-plane defined by $x \ge 0$, $-\infty < y < \infty$ and hence solve $\nabla^2 u = 0$ in the above region subject to the condition $u = f(y)$ on $x = 0$, using the Green's function technique.

19. Determine the Green's function for the Dirichlet problem for a circle given by $\nabla^2 u = 0$, $r < a$; $u = f(\theta)$ on $r = a$.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

20. Reduce the Tricomi equation $u_{xx} + xu_{yy} = 0$, $x \ne 0$ for all x, y to canonical form.

21. Discuss the interior Dirichlet problem for a circle and find its solution.

22. Determine the temperature in a sphere of radius a, when its surface is maintained at zero while its initial temperature is $f(r,\theta)$.

23. Find the periodic solution of one dimensional wave equation in cylindrical coordinates.

24. A string is stretched and fixed between two points (0,0) and (l, 0). Motion is initiated by displacing the string in the form $u = \lambda \sin(\pi x / l)$ and released from rest at time $t = 0$. Find the displacement of any point on the string at any time t.

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