DECEMBER 2024 **P/ID 37508/PMA2H**

Time : Three hours Maximum : 80 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Find the PDE by eliminating the arbitrary function from $z = f(x + it) + g(x - it)$, where $i=\sqrt{-1}$.

2. Write the nonlinear one-dimensional Burger equation.

3. Construct an adjoint to the Laplace operator $L(u)=u_{xx}+u_{yy}$.

4. Write the Neumann problem for a rectangle.

5. Write the solution of Laplace equation in cylindrical coordinates.

6. Write the three types of boundary conditions.

7. Write any two properties of Dirac delta function.

8. Write the Maxwell's equation of electromagnetic theory.

9. Write the periodic solution of one dimensional wave equation in spherical polar coordinates.

10. Write the Poisson's integral formula.

11. Write Helmholtz theorem.

12. Write the Green's function for heat transfer in infinite rod.

PART B — ($5 \times 6 = 30$ marks)

Answer any FIVE questions.

13. Find the complete integral of $x^2p^2 + y^2q^2 = 4$ using Charpit's method.

14. Derive the Poisson equation.

15. A one-dimensional infinite region $-\infty < x < \infty$ is initially kept at zero temperature. A heat source of strength g_s units, situated at $x=\xi$ releases its heat instantaneously at time $t=\tau$. Determine the Temperature in the region for $t>\tau$.

16. Determine the temperature $T(r,t)$ in the infinite cylinder $0 \le r \le a$ when the initial temperature is $T(r,0)=f(r)$ and the surface $r = a$ is maintained at 0° temperature.

17. Find the D'Alembert's solution of the one-dimensional wave equation.

18. Find the Green's function for the Dirichlet problem on the rectangle $\mathbb{R}$: $0 \le x \le a, 0 \le y \le b$, described by the PDE : $(\nabla^2 + \lambda)u = 0$ in $\mathbb{R}$ and the BC $u = 0$ on $\partial\mathbb{R}$.

19. Determine the Green's function for the Helmholtz equation for the half-space $z \ge 0$.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

20. Reduce the Tricomi equation $u_{xx} + xu_{yy} = 0$, $x \neq 0$ for all x, y to canonical form.

21. Solve the Poisson equation:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 2.$$

subject to the boundary conditions
$u(0, y) = u(5, y) = u(x, 0) = u(x, 4) = 0$.

22. A thin annulus occupies the region $0<a\le r\le b, 0\le\theta\le 2\pi$. The faces are insulated. Along the inner edge the temperature is maintained at 0°, while along the outer edge the temperature is held at $T = K \cos(\theta/2)$, where K is a constant. Determine the temperature distribution in the annulus.

23. Find the periodic solution of one dimensional wave equation in cylindrical coordinates.

24. Using Laplace transform, solve the following initial boundary value problem:

$$PDE: ku_t = u_{xx}, \quad 0<x<1, \quad 0<t<\infty$$
$$BCs: u(0,t), u(l,t) = g(t), \quad 0<t<\infty$$
$$IC: u(x,0) = 0, \quad 0<x<l.$$

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