MAY 2022 **P/ID 37508/PMA2H**

Time : Three hours Maximum : 80 marks

PART A — (10 × 2 = 20 marks)

Answer any TEN questions.

1. Form the partial differential equation by eliminating the arbitrary function from $z = f\left(\frac{xy}{z}\right)$.

2. Find the general integral of the following linear PDEs :

   $(x^2 - yz)\,p + (y^2 - zx)q = z^2 - xy$.

3. Find the complete integral of the $p^2 y(1 + x^2) = qx^2$.

4. Define the Laplace operator.

5. Write the Maxwell's equation.

6. Write the boundary condition II.

7. Prove that $\int_{-\infty}^{\infty} f(t)\delta(t)dt = f(0)$.

8. Prove that $\delta(-t) = \delta(t)$.

9. State uniqueness theorem.

10. Write the Neumann condition.

11. Write the Poisson integral formula.

12. Define a test functions.

PART B — ($5 \times 6 = 30$ marks)

Answer any FIVE questions.

13. Find the partial differential equation of the family of planes, the sum of whose x, y, z intercepts is equal to unity.

14. Find the general integral of the partial differential equation $(y + zx)\, p - (x + yz)\, q = x^2 - y^2$.

15. Find the potential at all points of space inside and outside of a sphere of radius $R = 1$ which is maintained at a constant distribution of electric potential $u(R, \theta) = f(\theta) = \cos 2\theta$.

16. A one dimensional infinite region $-\infty < x < \infty$ is initially kept at zero temperature. A heat source of strength g_s, situated at $x = \xi$ releases its heat instantaneously at time $t = T$. Determine the temperature in the region for $t > T$.

17. The ends A and B of a rod, 10 cm in length, are kept at 0°C and 100° until the steady state conditions prevails. Suddenly the temperature at the end A is increased to 20°C and the end B is decreased to 60°C. Find the temperature distribution in the rod at time t.

18. A string of length L is released from rest in the position $y = f(x)$. Show that the total energy of the string is $\frac{\pi^2 T}{4L}\sum_{n=1}^{\infty} s^2 k_s^2$ where $k_s = \frac{2}{L}\int_0^L f(x)\sin(s\pi x/L)dx$, T-tension in the string.

If the mid point of a string is pulled aside through a small distance and then released, show that in the subsequent motion the fundamental mode contributed $8/\pi^2$ of the total energy.

19. If G is continuous and $\partial G/\partial n$ has discontinuity at $\vec{r}$, show that $\lim_{\in \to 0}\iint_{\partial C}\frac{\partial G}{\partial n}ds = 1$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

20. Reduce the following equation to a canonical form and hence solve it.

$u_{xx} - 2\sin x u_{xy} - \cos^2 x u_{yy} - \cos x u_y = 0$.

21. A thin rectangular homogenous thermally conducting plate lies in the xy-plane defined by $0 \le x \le a, 0 \le y \le b$. The edge $y = 0$ is held at the temperature $Tx(x-a)$, where T is a constant, while the remaining edges are held at 0 degree. The other faces are insulated and no internal sources and sinks are present. Find the steady state temperature inside the plate.

22. A uniform rod of length L whose surface is thermally insulated is initially at temperature $\theta = \theta_0$. At time $t = 0$ one end is suddenly cooled to $\theta = 0$ and subsequently maintained at this temperature, the other end remains thermally insulated. Find the temperature distribution $\theta(x,t)$.

23. A infinite one dimensional solid defined by $-\infty < x < \infty$ is maintained at zero temperature initially. There is a heat source of strength $g_s(t)$ units, situated at $x = \xi$, which releases constant heat continuously for $t > 0$. Find an expression for the temperature distribution in the solid for $t > 0$.

24. Obtain the solution of the interior Dirichlet problem for a sphere using the Green function method and hence derive the Poisson integral formula.

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