MAY 2023 **P/ID 37508/PMA2H**

Time : Three hours Maximum : 80 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Find the PDE by eliminating the arbitrary function from $z = f(x+it)+g(x-it)$, where $i = \sqrt{-1}$.

2. Write the linear three-dimensional heat equation.

3. Find the characteristics of the equation $u_{xx} + 2u_{xy} + \sin^2(x)u_{yy} + u_y = 0$, when it is of hyperbolic type.

4. Write the Neumann problem for a rectangle.

5. Write the solution of Laplace equation in cylindrical coordinates.

6. Define the unit impulse function.

7. Write any two properties of Dirac delta function.

8. Write the D'Alembert's solution of the one dimensional wave equation.

9. Write the periodic solution of one dimensional wave equation in spherical polar coordinates.

10. Write the Poisson's integral formula.

11. Write the physical interpretation of Green's function.

12. If $u(x, t)$ is a function of two variables x and t, find $L\left[\frac{\partial u}{\partial x}; s\right]$.

PART B — ($5 \times 6 = 30$ marks)

Answer any FIVE questions.

13. Find the complete integral of $x^2p^2 + y^2q^2 - 4 = 0$ using Charpit's method.

14. Show that the velocity potential for an irrotational flow of an incompressible fluid satisfies the Laplace solution.

15. A uniform rod of length L whose surface is thermally insulated is initially at temperature $\theta = \theta_0$. At time $t = 0$, one end is suddenly cooled to $\theta = 0$ and subsequently maintained at this temperature ; the other end remains thermally insulated. Find the temperature distribution $\theta(x, t)$.

16. Determine the temperature $T(r, t)$ in the infinite cylinder $0 \le r \le a$ when the initial temperature is $T(r, 0) = f(r)$, and the surface $r = a$ is maintained at 0° temperature.

17. A stretched string of finite length L is held fixed at its ends and is subjected to an initial displacement $u(x, 0) = u_0 \sin(x\pi / L)$. The string is released from this position with zero initial velocity. Find the resultant time dependent motion of the string.

18. Show that the Green's function $G(r, r')$ has the symmetric property.

19. Determine the Green's function for the Helmholtz equation for the half-space $z \ge 0$.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

20. Find the Green's function for the equation $\dfrac{\partial^2 u}{\partial x \partial y} + u = 0.$

21. Find the potential u inside the cylinder $0 \le r \le a$, $0 \le \theta \le 2\pi$, $0 \le z < h$, if the potential on the top $z = h$ and on the lateral surface $r = a$ is held at zero, while on the base $z = 0$, the potential is given by $u(r, \theta, 0) = V_0\left(1 - r^2 / a^2\right)$, where V_0 is a constant ; r, θ, z are cylindrical polar coordinates.

22. A thin annulus occupies the region $0 < a \le r \le b$, $0 \le \theta \le 2\pi$. The faces are insulated. Along the inner edge, the temperature is maintained at 0°, while along the outer edge, the temperature is held at $T = K\cos(\theta/2)$, where K is a constant. Determine the temperature distribution in the annulus.

23. Prove that the total energy of a string, which is fixed at the points $x = 0$, $x = L$ and executing small transverse vibrations, is given by

$$\frac{1}{2}T\int_0^L\left[\left(\frac{\partial y}{\partial x}\right)^2 + \frac{1}{C^2}\left(\frac{\partial y}{\partial t}\right)^2\right]dx.$$

24. Obtain the solution of the interior Dirichlet problem for a sphere using the Green's function method and hence derive the Poisson's integral formula.

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