MAY 2024 **P/ID 37508/PMA2H**

Time : Three hours Maximum : 80 marks

PART A — $(10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Find the PDE by eliminating the arbitrary function from $z = f(xy/z)$.

2. Write the linear three-dimensional wave equation.

3. Write the adjoint to the Laplace operator $L(u) = u_{xx} + u_{yy}$.

4. Write the Dirichlet problem for a rectangle.

5. Write the solution of Laplace equation in cylindrical coordinates.

6. Write the three types of boundary conditions.

7. Write any two properties of Dirac delta function.

8. Write the periodic solution of one dimensional wave equation in spherical polar coordinates.

9. Write the Maxwell's equation of electromagnetic theory.

10. Write the physical interpretation of Green's function.

11. Define the Green's function for the Dirichlet problem involving the Laplacian operator.

12. Write the Green's function for heat transfer in infinite rod.

PART B — $(5 \times 6 = 30$ marks)

Answer any FIVE questions.

13. Find the complete integral of $x^2p^2 + y^2q^2 = 4$ using Charpit's method.

14. Derive the Laplace's equation.

15. A one-dimensional infinite region $\infty < x < \infty$ is initially kept at zero temperature. A heat source of strength g_s units, situated at $x = \xi$ releases its heat instantaneously at time $t = \tau$. Determine the temperature in the region for $t > \tau$.

16. Determine the temperature $T(r,t)$ in the infinite cylinder $0 \le r \le a$ when the initial temperature is $T(r,0) = f(r)$, and the surface $r = a$ is maintained at 0° temperature.

17. Obtain the solution of the wave equation $u_u = c^2\ u_{xx}$ under the conditions: $u(0,t) = u(2,t) = 0$; $u(x,0) = \sin^3 \pi x/2$; $u_t(x,0) = 0$.

18. Find the Green's function for the Dirichlet problem on the rectangle $\mathbb{R}$: $0 \le x \le a$, $0 \le y \le b$, described by the PDE: $(\nabla^2 + \lambda)u = 0$ in $\mathbb{R}$ and the BC $u = 0$ on $\partial\,\mathbb{R}$.

19. If G is continuous and $\partial G/\partial n$ has discontinuity at r, in particular show that

$$\lim_{\varepsilon \to 0} \iint_{\partial G} \frac{\partial G}{\partial n}\, dS = 1.$$

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

20. Find the complete integral of $(p^2 + q^2)\, y = qz$.

21. Solve the Poisson equation:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 2.$$

Subject to thewboundary conditions $u(0, y) = u(5, y) = u(x,0) = u(x,4) = 0$.

22. The ends A and B of a rod, 10 cm in length, are kept at temperatures 0° C and 100°C until the steady state condition prevails. Suddenly the temperature at the end A is increased to 20° C and the end B is decreased to 60° C. Find the temperature distribution in the rod at time t.

23. Solve the initial value problem of the wave equation described by the homogeneous wave equation

$$\text{PDE: } u_{tt} - c^2 u_{xx} = f(x,t)$$

subject to the initial conditions $u(x,0) = \eta(x)$, $u_t(x,0) = \upsilon(x)$.

24. Using Laplace transform, solve the following.

PDE: $ku_t = u_{xx}, 0 < x < 1, 0 < t < \infty$

BCs: $u(0,t), u(l,t) = g(t),\ 0 < t < \infty$

IC: $u(x,0) = 0,\ 0 < x < l$.

———————