MAY 2025 **P/ID 37508/PMA2H**

Time : Three hours Maximum : 80 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Find the PDE by eliminating the arbitrary function from $z = xy + f(x^2 + y^2)$.

2. Write the linear three-dimensional wave equation.

3. Find the characteristics of the equation $u_{xx} + 2u_{xy} + \sin^2(x)u_{yy} + u_y = 0$, when it is of hyperbolic type.

4. Write the exterior Dirichlet problem for a circle.

5. Write the solution of Laplace equation in cylindrical coordinates.

6. Define the unit impulse function.

7. Write any two properties of Dirac delta function.

8. Write Robin's condition.

9. Write the periodic solution of one dimensional wave equation in spherical polar coordinates.

10. Write the physical interpretation of Green's function.

11. Define the Green's function for the Dirichlet problem involving the Laplacian operator.

12. If $u(x,t)$ is a function of two variables x and t, find $L\left[\frac{\partial u}{\partial x};s\right]$.

PART B — ($5 \times 6 = 30$ marks)

Answer any FIVE questions.

13. Find the characteristics of the PDE : $p^2 + q^2 = 2$ and determine the integral surface which passes through $x = 0, z = y$.

14. Derive the Laplace's equation.

15. A uniform rod of length L whose surface is thermally insulated is initially at temperature $\theta = \theta_0$. At time $t = 0$, one end is suddenly cooled to $\theta = 0$ and subsequently maintained at this temperature; the other end remains thermally insulated. Find the temperature distribution $\theta(x,t)$.

16. Determine the temperature $T(r,t)$ in the infinite cylinder $0 \le r \le a$ when the initial temperature is $T(r,0) = f(r)$, and the surface $r = a$ is maintained at 0° temperature.

17. Obtain the solution of the wave equation $u_{tt} = c^2 u_{xx}$ under the conditions :

$$u(0,t) = u(2,t) = 0;\ u(x,0) = \sin^3 \frac{\pi x}{2};\ u_t(x,0) = 0 .$$

18. Show that the Green's function $G(r,r')$ has the symmetric property.

19. Determine the Green's function for the Dirichlet problem for a circle given by $\nabla^2 u = 0, r < a$; $u = f(\theta)$ on $r = a$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

20. Find the complete integral of $(p^2 + q^2)y = qz$.

21. Find the potential u inside the cylinder $0 \le r \le a, 0 \le \theta \le 2\pi, 0 \le z < h$, if the potential on the top $z = h$ and on the lateral surface $r = a$ is held at zero, while on the base $z = 0$, the potential is given by $u(r,\theta,0) = V_0(1 - \frac{r^2}{a^2})$, where V_0 is a constant; r, θ, z are cylindrical polar coordinates.

22. Determine the temperature in a sphere of radius a, when its surface is maintained at zero while its initial temperature is $f(r,\theta)$.

23. Solve the initial value problem of the wave equation described by the homogeneous wave equation

$$PDE: u_{tt} - c^2\ u_{xx} = f(x,t)$$

subject to the initial conditions $u(x,\ 0) = \eta(x),\ u_t(x,\ 0) = \upsilon(x)$.

24. Obtain the solution of the interior Dirichlet problem for a sphere using the Green's function method and hence derive the Poisson's integral formula.

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