DECEMBER 2022 **P/ID 37509/PMA2J**

Time : Three hours Maximum : 80 marks

SECTION A — (10 × 2 = 20 marks)

Answer any TEN questions.

1. What is the order of convergence of Newton Raphson method?

2. Explain Gauss elimination method to solve $AX = B$.

3. When Newton's backward interpolation formula is used?

4. Mention any two uses of Lagrange's formula.

5. List out the differences between Gauss elimination and Gauss Seidal methods.

6. Write the formula for finding the first derivative using forward difference formula.

7. Write the three-point midpoint formula for first derivative.

8. Write trapezoidal rule to evaluate $\int_{x_0}^{x_n} y(x)\,dx$.

9. Define Simpson's 1/3-rule.

10. Define Euler's method.

11. What is the error term in Taylor's method of order four.

12. Define classical fourth order Runge-Kutta method.

SECTION B — ($5 \times 6 = 30$ marks)

Answer any FIVE questions.

13. Find a positive root of $x^3 + 3x - 1 = 0$ by using Newton's method.

14. Given that $f(0) = 1$, $f(1) = 3$, $f(3) = 5$, find the Lagrange interpolating polynomial of degree two or less which fits the given data.

15. Use the forward difference method with h = 0.1 and h = 0.05 to find the approximation to $f'(1.8)$ for $f(x) = \ln(x)$. Assume that this formula has truncation error $O(h)$ and use extrapolation on these values to obtain a better approximation.

16. Using second derivative approximation formula, approximate $f''(2.0)$ using the given table of values for $f(x) = x\,e^x$ taking h = 0.1 and compute its actual error.

x	1.8	1.9	2.0	2.1	2.2
$f(x)$	10.889365	12.703199	14.778112	17.148957	19.859030

17. Evaluate $\int_0^{\pi} x^2 \cos x dx$ using composite trapezoidal rule and composite Simpson's rule by taking n = 6.

18. Use Euler's method to approximate the solution to the initial value problem

$$y' = y - t^2 + 1,\ 0 \le t \le 2,\ y(0) = 0$$

with h = 0.5.

19. Use Euler's method to approximate the solution to the initial value problem

$$y' = t + y,\ 0 \le t \le 1,\ y(0) = 1$$

with h = 0.2.

SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

20. Find a root of $x \log_{10} x - 1.2 = 0$ using bisection method and correct to three decimal places.

21. Solve the system by Gauss elimination method

$$\begin{aligned} x_1 + x_2 + 3x_4 &= 4 \\ 2x_1 + x_2 - x_3 + x_4 &= 1 \\ 3x_1 - x_2 - x_3 + 2x_4 &= -3 \\ -x_1 + 2x_2 + 3x_3 - x_4 &= 4 \end{aligned}$$

22. Use the forward-difference formula and backward-difference formula to determine the first derivative of each entry in the following table

x	0.5	0.6	0.7
$f(x)$	0.4794	0.5646	0.6442

23. Dividing the range into 10 equal parts, find the approximate value of $\int_0^{\pi} \sin x dx$ using

(a) Composite Trapezoidal rule and

(b) Composite Simpson's rule.

Also evaluate the actual integral.

24. Using Runge-Kutta method of order four, find the solution to the initial value problem $y' = ty + t^2,\ y(0) = 1 \qquad 0 \le t \le 0.3$. Continue the solution by Milne-Simpson Predictor-Corrector method to approximate $y(0.4)$.

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