DECEMBER 2023 **P/ID 37509/PMA2J**

Time : Three hours Maximum : 80 marks

SECTION A — ($10 \times 2 = 20$ marks)

Answer any TEN questions

1. Write the formula of the Newton Raphson Method.

2. State the Fixed point theorem.

3. Define spectral radius.

4. Define convergent of a matrix.

5. State three point formula.

6. State five point mid point formula.

7. Find sum of Simpson's rule approximation for $\int_0^2 e^x dx$.

8. State composite Simpsons rule.

9. State Euler's method.

10. State Milne Predictor method.

11. Write down the formula for finding an approximate root of $f(x) = 0$ using secant method.

12. Write Lagrang's interpolation.

SECTION B — (5 × 6 = 30 marks)

Answer any FIVE questions

13. Use method of false position to find the solution to $xe^x - \cos x = 0$.

14. Find the positive root of $\cos x - x = 0$ by using Newton Raphson's method.

15. Use the nodes $x_0 = 2, x_1 = 2.75$ and $x_2 = 4$ to find the second Lagrange interpolating polynomial for $f(x) = 1/x$.

16. Use the forward difference formula to approximate the derivative of $f(x) = ln(x)$ at $x_0 = 1.8$ using $h = 0.1$ and determine bounds for the approximation error.

17. Determine value of k that will ensure an approximation error of less that 0.00002 when approximating $\int_0^{\pi} \sin x \, dx$ by composite Trapezoidal rule.

18. Use Euler's method to approximate the solutions for each of the following initial value problems $y' = t\ e^{3t} - 2y;\ 0 \le t \le 1,\ y(0) = 0,$ with $h = 0.5$.

19. Apply Taylors method of order 2 with N = 10 to the solution of the initial value problem $y' = y - t^2 + 1,\ 0 \le t \le 2,\ y(0) = 0.5$.

SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions

20. Show that $f(x) = x^3 + 4x^2 - 10 = 0$ has a root in $[1, 2]$ and use the Bisection method to determine an approximation to the root that is accurate to at least within 10^{-4}.

21. Solve the following system of equations by Gauss Seidel method.

$2x + 10y + z = 13,\ 10x + 2y + z = 13,\ 2x + y + 10z = 13$.

22. Apply the extrapolation process to determine $N_3(0.2)$, an approximation to $f'(2.0)$, for $f(xe^x)$ and step size $h = 0.2$.

23. Use the composite Trapezoidal rule with the indicated value of n to approximate the following integrals.

(a) $\int_{-2}^{2} x^3 e^x \, dx, \; n = 4$

(b) $\int_{0}^{2} \frac{2}{x^2 + 4} dx, \; n = 6$

24. Use Taylor's method of order two to approximate the solutions for each of the following initial value problems.

$y' = \cos 2t + \sin 3t, \; 0 \le t \le 1, \text{ with } h = 0.25$.