DECEMBER 2024 **P/ID 37509/PMA2J**

Time : Three hours Maximum : 80 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Define order of convergence.

2. Write any two merits of Bisection Method.

3. Define convergence of a matrix

4. Define spectral radius.

5. Explain Differentiation using extrapolation.

6. Write the formula for forward difference derivative.

7. Write composite Simpson's rule.

8. Compute sum of Simpson's rule approximation for $\int_0^2 e^x dx$.

9. Write the disadvantages of the Taylor's method.

10. Write Euler's method.

11. Write Milne-Predictor method.

12. Define fixed point of a function.

PART B — (5 × 6 = 30 marks)

Answer any FIVE questions.

13. Suppose that $f \in C[a,b]$ and $f(a) \cdot f(b) < 0$. Then prove that the Bisection method generates a sequence $\{p_n\}_{n=1}^{\infty}$ approximating a zero p of f with $|p_n - p| \leq \frac{b-a}{2^n}$, when $n \geq 1$.

14. Solve the following system or equations by Gauss elimination method.

$3x + 4y - z = 8$, $-2x + y + z = 3$, $x + 2y - z = 2$

15. Find the positive root of $\cos x - x = 0$ by Newton-Raphson's method.

16. Use the forward-difference formula to approximate the derivative of $f(x) = \ln(x)$ at $x_0 = 1.8$ using $h = 0.1$ and determine bounds for the approximation error.

17. Use Euler's method to approximate the solutions of the initial value problem $y' = te^{3t} - 2y$, $0 \le t \le 1$, $y(0) = 0$, with $h = 0.5$.

18. Determine the value of h that will ensure an approximate error of less than that of 0.00002 when approximating $\int_0^{\pi} \sin x dx$ using composite Simpson's rule.

19. Apply Taylor's method of order 2 with $N = 10$ to the initial value problem

$y' = y - t^2 + 1$, $0 \le t \le 2$, $y(0) = 0.5$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

20. Let $f \in C^2[a,b]$. If $p \in (a,b)$ is such that $f(p) = 0$ and $f'(p) \neq 0$. Then prove that there exists $\delta > 0$ such that Newton's method generates a sequence $\{p_n\}_{n=1}^{\infty}$ converging to p for any initial approximation $p_0 \in [p - \delta, p + \delta]$.

21. Solve the following system of equations by Gauss Seidel method.

$8x + y + z = 8$, $2x + 4y + z = 4$, $x + 3y + 5z = 5$

22. Derive the second derivative midpoint formula.

23. Use the composite Trapezoidal rule with the indicated value of n to approximate the following integrals.

(a) $\int_{-2}^{2} x^3 e^x dx, \quad n = 4$

(b) $\int_{0}^{2} \frac{2}{x^2 + 4} dx, \quad n = 6$

24. Use Taylor's method of order two to approximate the solutions for the initial value problem

$y' = 1 + \frac{y}{t}, \quad 1 \le t \le 2$, with $h = 0.25$.