MAY 2022 **P/ID 37509/PMA2J**

Time : Three hours Maximum : 80 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Write the merits of bisection method.
2. Write the Newton Raphson's iteration for finding the approximations of $\sqrt{n}$.
3. What is inverse interpolation?
4. Write Gauss elimination process.
5. What is numerical integration?
6. Write the five point mid value formula for first derivative.
7. Define spectral radius.
8. Write Trapezodial rule.
9. Why Simpson's 1/3-rule is called a closed formula?
10. What is the error term in Taylor's method of order two?
11. Write Milne's Predictor formula.
12. Write the Euler's formula.

PART B — ($5 \times 6 = 30$ marks)

Answer any FIVE questions.

13. Use Regula-Flasi method to find a solution to $x = \cos x$.

14. Use Gauss-Seidal iterative technique to find approximate solutions to

$$\begin{aligned} 10x_1 - 2x_2 - x_3 - x_4 &= 3 \\ -2x_1 + 10x_2 - x_3 - x_4 &= 15 \\ -x_1 - x_2 + 10x_3 - 2x_4 &= 27 \\ -x_1 - x_2 - 2x_3 + 10x_4 &= -9 \end{aligned}$$

15. Using second derivative approximation formula, approximate $f''(2.0)$ using the given table of values for $f(x) = xe^x$ taking $h = 0.1$ and compute its actual error.

x :	1.8	1.9	2.0	2.1	2.2
$f(x)$:	10.889365	12.703199	14.778112	17.148957	19.859030

16. Use the forward difference method with $h = 0.1$ and $h = 0.05$ to find the approximation to $f'(1.8)$ for $f(x) = \ln(x)$. Assume that this formula has truncation error $O(h)$ and use extrapolation on these values to obtain a better approximation.

17. Evaluate $\int_{-2}^{2} x^3 e^x dx$ using

(a) composite trapezoidal rule taking $n = 4$; and

(b) composite Simpson's rule taking $n = 4$.

18. Use Taylor's method to find $y(0.1)$, given $y' = t^2 + y^2$, $y(0) = 1$.

19. Use Runge Kutta method of order four to obtain the solution of the initial value problem $y' = y - t^2 + 1$, $y(0) = 0.5$ taking $h = 0.2$.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

20. Show that $f(x) = x^3 + 4x^2 - 10 = 0$ has a root in [1, 2] and use the bisection method to determine an approximation to the root that is accurate to atleast within 10^{-4}.

21. Solve the system by Gauss elimination method.

$$\begin{aligned} x_1 + x_2 + \quad & + 3x_4 = 4 \\ 2x_1 + x_2 - x_3 + x_4 \quad & = 1 \\ 3x_1 - x_2 - x_3 + 2x_4 & = -3 \\ -x_1 + 2x_2 + 3x_3 - x_4 & = 4 \end{aligned}$$

22. Use the most accurate three-point formula to determine first derivative at each of the points in the following table.

x :	1.1	1.2	1.3	1.4
$f(x)$:	9.025013	11.02318	13.46374	16.4465

23. (a) Evaluate $\int_0^4 e^x dx$ using Simpson's rule.

(b) Compute sum of Simpson's rule approximations for $\int_0^2 e^x dx$ and $\int_2^4 e^x dx$.

(c) Compute sum of Simpson's rule approximation for $\int_0^1 e^x dx, \int_1^2 e^x dx, \int_2^3 e^x dx$ and $\int_3^4 e^x dx$.

(d) Compute the actual integral.

24. Use Euler's method to approximate the solution of the initial value problem

$$y' = y - t^2 + 1,\ 0 \le t \le 2,\ y(0) = 0.5$$

with $h = 0.2$, given the exact solution $y = (t+1)^2 - 0.5e^t$. Compare the actual errors.

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