MAY 2023 **P/ID 37509/PMA2J**

Time : Three hours Maximum : 80 marks

SECTION A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Write any two merits of Bisection Method.

2. Define order of convergence.

3. Define an eigen value.

4. What is the rate of convergence in Gauss elimination method?

5. Write the forward difference derivative formula.

6. Write the rate of convergence of Newton Raphson's method.

7. Evaluate $\int_0^2 e^x dx$.

8. State composite Simpson's rule.

9. Explain the error bounds for Eulers method.

10. Write the disadvantage of the Taylor's method.

11. Write down the formula for finding an approximate root of $f(x) = 0$ using secant method.

12. Write Lagrange's interpolation formula.

SECTION B — ($5 \times 6 = 30$ marks)

Answer any FIVE questions.

13. Use method of false position to find the solution of $x^2 - 2x - 5 = 0$.

14. Use the secant method to find a solution of $x = \cos x$.

15. Use the numbers $x_0 = 2, x_1 = 2.75$ and $x_2 = 4$ to find the second Lagrange interpolating polynomial for $f(x) = 1/x$.

16. Derive the forward difference derivative formula.

17. Determine the value of h that will ensure an approximation error of less that 0.00002 when approximating $\int_0^{\pi} \sin x \, dx$ by composite Simpson's rule.

18. Use Euler's method to approximate the solutions for each of the following initial value problems $y' = 1 + (t - y)^2;\ 2 \le t \le 3,\ y(2) = 1,$ with $h = 0.5$.

19. Use the Runge Kutta method of order four with $h = 0.2,\ N = 10$ and $t_i = 0.2\,i$ to obtain approximation to the solution of the initial value problem $y' = y - t^2 + 1,\ 0 \le t \le 2,\ y(0) = 0.5$.

SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

20. Let $f \in C^2[a, b]$. If $p \in (a, b)$ is such that $f(p) = 0$ and $f'(p) \ne 0,$ then prove that there exist $a\,\delta > 0$ such that Newton's method generates a sequence $\{p_n\}_{n+1}^{\infty}$ converging to p for any initial approximation $p_{ij} \in [p - \delta, p + \delta]$.

21. Solve the following system of equations by Gauss Seidal method.

$12x + 3y - 5z = 1,\ x + 5y + 3z = 28,$

$3x + 7y + 13z = 76.$

22. Derive the second derivative midpoint formula.

23. Use the composite Simpson's rule with the indicated value of n to approximate the following integral :

(a) $\int_0^{\pi} x^2 \cos x dx,\ n = 6$,

(b) $\int_0^{\pi} e^{2x} \sin 3x\ dx,\ n = 8$.

24. Use Taylor's method of order two to approximate the solutions for each of the following initial value problems

$y' = 1 + y/t, 1 \le t \le 2$ with $h = 0.25$.