MAY 2024 **P/ID 37509/PMA2J**

Time : Three hours Maximum : 80 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Write any two demerits of Bisection Method.

2. Define Fixed point of a function.

3. Define spectra radius.

4. Write the Lagrange interpolation polynomial through (x_0, y_0) and (x_1, y_1).

5. Write the forward difference derivative formula.

6. Explain Differentiation using extrapolation.

7. Evaluate $\int_0^4 e^x \, dx$ using composite Simpson's rule.

8. State composite Trapezoidal rule.

9. State Eulers method.

10. State Milne Predictor method.

11. Write any two merits of bisection method.

12. Write Lagrang's interpolation.

PART B — ($5 \times 6 = 30$ marks)

Answer any FIVE questions.

13. Suppose $f \in C[a,b]$ and $f(a) \cdot f(b) < 0$. Then prove that the Bisection method generates a sequence $\{p_n\}_{n=1}^{\infty}$ approximating a zero p of f with $|p_n - p| \leq \frac{b-a}{2^n}$ when $n \geq 1$.

14. Find the positive root of $\cos x - x = 0$ by using Newton Raphson's method.

15. Solve the following system of equations by Gauss elimination method: $3x + 4y - z = 8; -2x + y + z = 3;$ $x + 2y - z = 2$.

16. Use the forward difference formula and backward difference formula to find the first derivative at each entry in the following table:

x	0.5	0.6	0.7
$f(x)$	0.4794	0.5646	0.6442

17. Determine value of h that will ensure an approximation error of less that 0.00002 when approximating $\int_0^{\pi} \sin x \, dx$ by composite Simpson's rule.

18. Use Euler's method to approximate the solutions for each of the following initial value problems $y' = 1 + (t - y)^2 ; 2 \le t \le 3, y(2) = 1,$ with $h = 0.5$.

19. Apply Taylors method of order 2 with $N = 10$ to the solution of the initial value problem $y' = y - t^2 + 1, 0 \le t \le 2, y(0) = 0.5$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

20. Show that $f(x) = x^3 + 4x^2 - 10 = 0$ has a root in $[1,2]$ and use the Bisection method to determine an approximation to the root that is accurate to at least within 10^{-4}.

21. Solve the following system of equations by Gauss Seidel method. $8x + y + z = 8, 2x + 4y + z = 4,$ $x + 3y + 5z = 5$.

22. Approximate $f'(2.0)$ using the given table of values for $f(x) = xe^x$ taking $h = 0.1$ using (a) Three point End point formula (b) Three point Midpoint formula and (c) Five point Midpoint formula.

23. Use the composite Trapezoidal rule with the indicated value of n to approximate the following integral (a) $\int_{-2}^{2} x^3 e^x \, dx, n = 4$, (b) $\int_{0}^{2} \frac{2}{x^2 + 4} dx, n = 6$.

24. Use Taylor's method of order two to approximate the solutions for each of the following initial value problems $y' = 1 + y/t, 1 \le t \le 2$, with $h = 0.25$.