MAY 2025 **P/ID 37509/PMA2J**

Time : Three hours Maximum : 80 marks

PART A — (10 × 2 = 20 marks)

Answer any TEN questions.

1. Define Fixed point of a function.

2. Write any two demerits of Bisection Method.

3. Write the Lagrange Interpolation polynomial through (x_0, y_0) and (x_1, y_1).

4. Define eigen values.

5. Explain Differentiation using extrapolation.

6. Write the formula for forward difference derivative.

7. Write Composite Trapezoidal rule.

8. Evaluate $\int_0^4 e^x \, dx$ using composite Simpson's rule.

9. Write the disadvantage of the Taylor's method.

10. Write Euler's method.

11. Define order of convergence.

12. Write Lagrange's interpolation formula.

PART B — ($5 \times 6 = 30$ marks)

Answer any FIVE questions.

13. Using the method of false position, find the solution of $x e^x - \cos x = 0$.

14. Using the nodes $x_0 = 2$, $x_1 = 2.75$ and $x_2 = 4$, find the second Lagrange interpolating polynomial for $f(x) = \frac{1}{x}$.

15. Use the Secant method to find a solution of $x = \cos x$.

16. Determine values of h that will ensure an approximation error of less that 0.00002 when approximating $\int_0^x \sin x \, dx$ by composite Simpson's rule.

17. Use the forward-difference formula and backward-difference formula to determine the first derivative of at each entry in the following table.

x	0.5	0.6	0.7
$f(x)$	0.4794	0.5646	0.6442

18. Use the Runge-Kutta method of order four with $h = 0.2$, $N = 10$ and $t_i = 0.2i$ to obtain approximations to the solution of the initial value problem

$y' = y - t^2 + 1,\ 0 \le t \le 2,\ y(0) = 0.5.$

19. Use Euler's method to approximate the solutions for the initial value problem $y' = 1 + (t - y)^2,\ 2 \le t \le 3,\ y(2) = 1$, with h =0.5.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

20. Let $f \in C^2[a, b]$. If $p \in (a, b)$ is such that $f(p) = 0$ and $f'(p) \neq 0$, then prove that there exists a $\delta > 0$ such that Newton's method generates a sequence $\{p_n\}_{n=1}^{\infty}$ converging to p for any initial approximation $p_0 \in [p - \delta,\ p + \delta]$.

21. Solve the following system of equations by Gauss Seidel method.

$5x - y + z = 10,\ 2x + 8y - z = 11,\ -x + y + 4z = 3.$

22. Approximate $f'(2.0)$ using the given table of values for $f(x) = x e^x$ taking $h = 0.1$ using

(a) Three point End point formula

(b) Three point Midpoint formula and

(c) Five point Midpoint formula.

23. Use the composite Simpson's rule with the indicated values of n to approximate the following integrals.

(a) $\int_0^{\pi} x^2 \cos x \ dx,\ n = 6$

(b) $\int_0^{2} e^{2x} \sin 3x \ dx,\ n = 8$

24. Use Taylors method of order two to approximate the solutions for the initial Value problem $y' = \cos 2t + \sin 3t,\ 0 \le t \le 1$ with $h = 0.25$.
