DECEMBER 2022 **P/ID 37511/PMA3L**

Time : Three hours Maximum : 80 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. State Cauchy's inequality.

2. Define meromorphic function.

3. Define locally exact differential.

4. State the residue theorem.

5. Find the poles of $\frac{1}{z^2 + 5z + 6}$.

6. Write the Poisson's formula in polar coordinates.

7. State Weierstrass theorem.

8. Write Laurent's series expansion of $f(z)$.

9. Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$.

10. State Jensen's formula.

11. Define modules.

12. Define gamma function.

PART B — ($5 \times 6 = 30$ marks)

Answer any FIVE questions.

13. State and prove Liouville's theorem.

14. State and prove the argument principle.

15. If u_1 and u_2 are harmonic in a region Ω, then prove that $\int_\gamma u_1 * du_2 - u_2 * du_1 = 0$.

16. State and prove Hurwitz theorem.

17. Prove that $\pi \cot \pi z = \frac{1}{z} + \sum_{n=1}^{\infty} \frac{2z}{z^2 - n^2}$.

18. Prove that $\Gamma(z)\Gamma(1-z) = \frac{\pi}{\sin \pi z}$.

19. Write down the Taylor's series of the following functions.

    (a) e^z

    (b) $\cos z$

    (c) $\log(1+z)$

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

20. Suppose that $\phi(\zeta)$ is continuous and the arc γ. Then prove that the function $F_n(z) = \int_\gamma \frac{\phi(\zeta)\,d\zeta}{(\zeta - z)^n}$ is analytic in each of the regions determined by γ, and its derivative $F'_n(z) = nF_{n+1}(z)$.

21. State and prove Rouche's theorem.

22. Derive Poisson's formula in Cartesian coordinates.

23. Prove that the arithmetic mean of a harmonic function over the concentric circles $|z| = r$ is a linear function of $\log r$, $\frac{1}{2\pi}\int_{|z|=r} u\,d\theta = \alpha \log r + \beta$ and if u is a harmonic in a disc $\alpha = 0$ and the arithmetic mean is a constant.

24. Obtain Legendre's duplication formula.

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