DECEMBER 2023 **P/ID 37511/PMA3L**

Time : Three hours Maximum : 80 marks

PART A — (10 × 2 = 20 marks)

Answer any TEN questions.

1. Define index of a point with respect to γ.

2. Define pole.

3. Define a chain.

4. Define residue.

5. Write the conjugate differential of du.

6. Find the poles and residues of cot z.

7. State reflection principle.

8. Write the Poisson integral of U.

9. Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$.

10. Does the product $(1+z)(1+z^2)(1+z^4)(1+z^8)\ldots$ converges? Justify your answer.

11. Define meromorphic function.

12. Is the series $1 + z + \frac{z^2}{2!} + ...+ \frac{z^n}{n!} + ...$ converges? Justify your answer.

PART B — ($5 \times 6 = 30$ marks)

Answer any FIVE questions.

13. State and prove Liouville's theorem.

14. State and prove Weierstrass theorem.

15. Prove that the arithmetic mean of a harmonic function over the concentric circles $|z| = r$ is a linear function of log r, $\frac{1}{2\pi}\int_{|z|=r} u\, d\theta = \alpha \log r + \beta$ and if u is harmonic in a disc $\alpha = 0$ and the arithmetic mean is a constant.

16. State and prove Hurwitz theorem.

17. Obtain the necessary and sufficient condition for the absolute convergence of the product $\prod_{1}^{\infty}(1+a_n)$.

18. State and prove Cauchy's theorem for higher derivatives.

19. State and prove residue theorem.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

20. Suppose that $\phi(\zeta)$ is continuous on the arc γ. Then prove that the function $F_n(z) = \int_\gamma \frac{\phi(\zeta)\, d\zeta}{(\zeta - z)^n}$ is analytic in each of the regions determined by γ, and its derivative $F_n'(z) = n\, F_{n+1}(z)$.

21. State and prove the general form of Cauchy's theorem.

22. Derive Poisson-Jensen's formula.

23. State and prove Schwarz theorem.

24. Obtain Legendre's duplication formula.

———————