DECEMBER 2024 **P/ID 37511/PMA3L**

Time : Three hours Maximum : 80 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Define index of a point a with respect to the closed curve γ.

2. Define pole.

3. Define Chain.

4. Define simply connected region.

5. Define harmonic function.

6. Find the poles of $\frac{1}{x^2+5z+6}$.

7. State reflection principle.

8. State Hurwitz theorem.

9. Define infinite products.

10. Show that $\prod_{n=2}^{\infty}\left(1-\frac{1}{n^2}\right)=\frac{1}{2}$.

11. Define residue.

12. Write conjugate differential of du.

PART B — ($5 \times 6 = 30$ marks)

Answer any FIVE questions.

13. If the piecewise differentiable closed curve γ does not pass through the point a, then prove that the value of the integral $\int_{\gamma}\frac{dz}{z-a}$ is a multiple of $2\pi i$.

14. State and prove Maximum principle.

15. State and prove Cauchy's Residue theorem.

16. How many roots does the equation $z^{\gamma}-2x^5+6z^3-z+1=0$ have in the disk $|z|<1$?

17. If u_1 and u_2 are harmonic in a region Ω, then prove that $\int_{\gamma}u_1 * du_2-u_2 * du_1=0$ for every cycle γ which is homologous to zero in Ω.

18. State and prove Schwarz theorem.

19. Prove that $\frac{\pi^2}{\sin^2 \pi z} = \sum_{-\infty}^{\infty} \frac{1}{(z-n)^2}$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

20. State and prove Taylor's theorem.

21. State and prove Cauchy's theorem.

22. Prove that the arithmetic mean of a harmonic function over the concentric circles $|z|=r$ is a linear function of $\log r$, $\frac{1}{2\pi}\int_{|z|=r} u\, d\theta = \alpha \log r + \beta$, and if u is harmonic in a disk $\alpha = 0$ and the arithmetic mean is constant.

23. Derive Laurent's series.

24. Obtain a necessary and sufficient condition for the absolute convergence of the product $\prod_{1}^{\infty}(1+a_n)$ is the convergence of the series $\sum_{1}^{\infty}|a_n|$.

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